FAST CORRECTOR MAGNET STUDIES: OVERVIEW AND EFFORTS TOWARDS NONLINEAR SIMULATIONS

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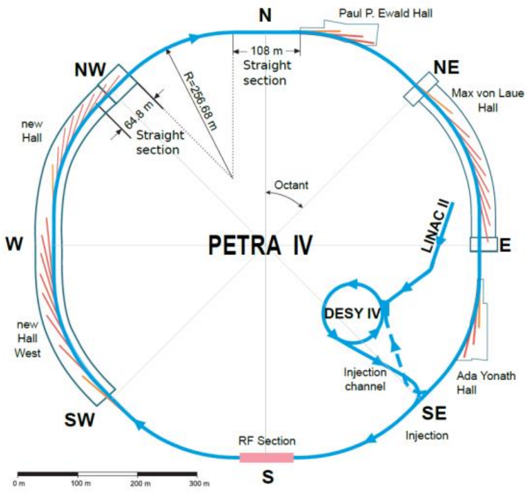
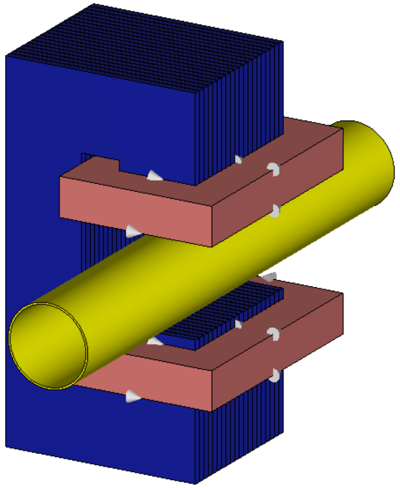
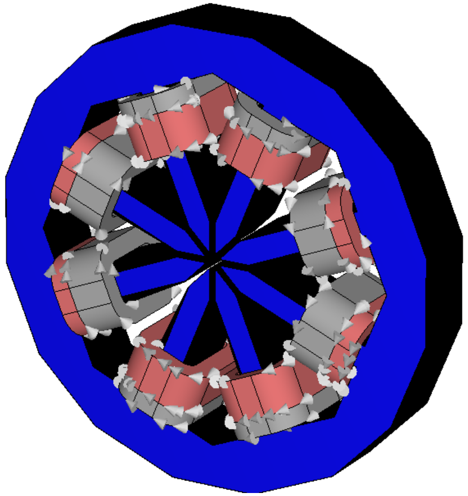
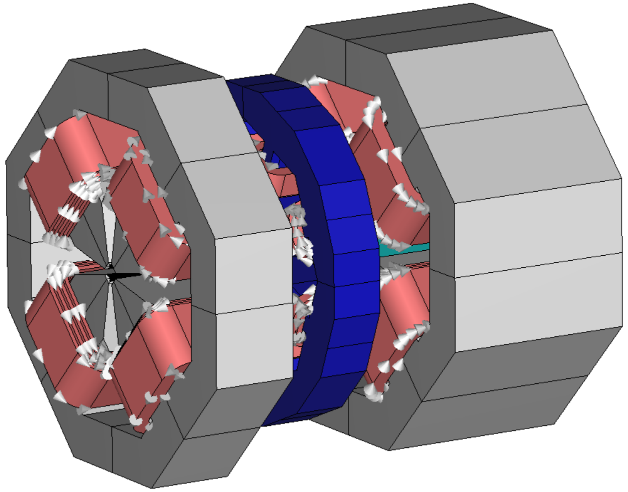


PETRA IV Conceptual Design Report







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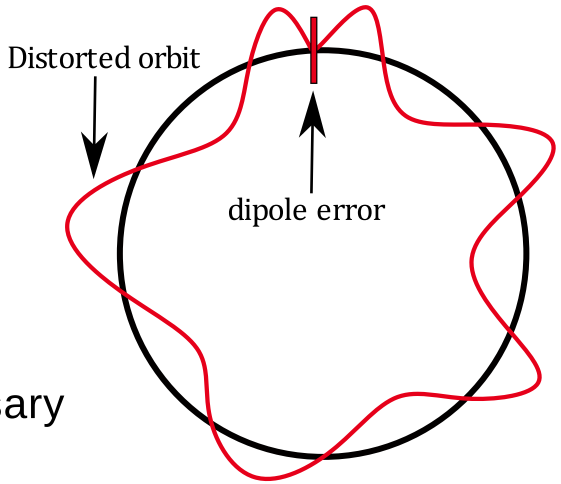
INTRODUCTION

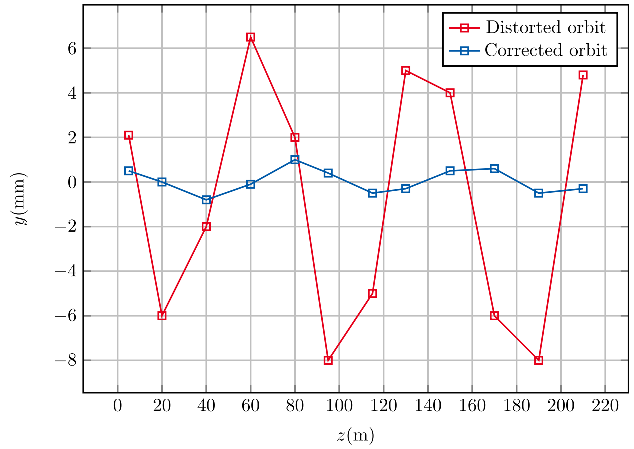


- Circular accelerators need dipole magnets to correct orbit distortions
- **PETRA IV**: ultra-low emittance synchrotron radiation source

➔ Fast orbit feedback system, **corrector magnets with frequencies in kHz range** necessary

- **Strong eddy currents ➔** power losses, time delay, and field distortion
- **Simulation challenging** due to small skin depths and laminated yoke

➔ Need for technique to simplify simulations



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THEORY

- Magnetoquasistatic PDE: $\nabla \times (\nu(\vec{r}\,)\nabla \times \underline{\vec{A}}\,(\vec{r})) + j\omega\sigma(\vec{r})\underline{\vec{A}}(\vec{r}) = \underline{\vec{J}_s}(\vec{r})$
- Replace reluctivity $\nu(\vec{r})$ and conductivity $\sigma(\vec{r})$ in the laminated yoke with spatially constant tensors

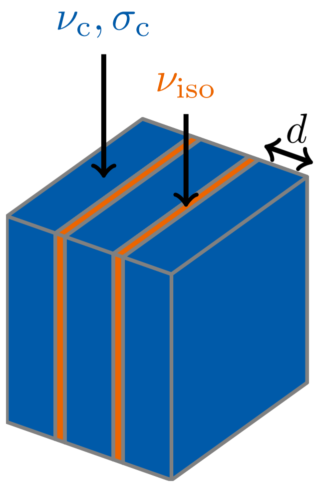
$$\nu(\vec{r}) \rightarrow \underline{\bar{\bar{\nu}}} = \frac{1}{8}\sigma_c d\delta\omega(1+j)\frac{\sinh((1+j)\delta^{-1}d)}{\sinh^2\big((1+j)\delta^{-1}\,d/2\big)}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix} + \nu_c\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

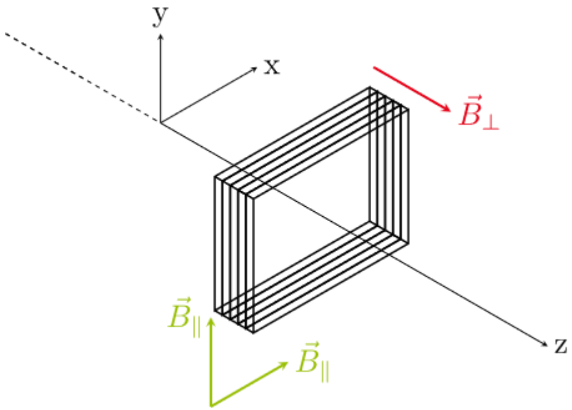
$$\sigma(\vec{r}) \rightarrow \bar{\bar{\sigma}} = \gamma\sigma_c\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Skin depth $\delta = \sqrt{2/\omega\sigma_c\mu_c}$

Stacking factor $\gamma = \frac{V_c}{V_{\text{Yoke}}}$

P. Dular et al., 2003
L. Krähenbühl et al., 2004
H. De Gersem et al., 2012





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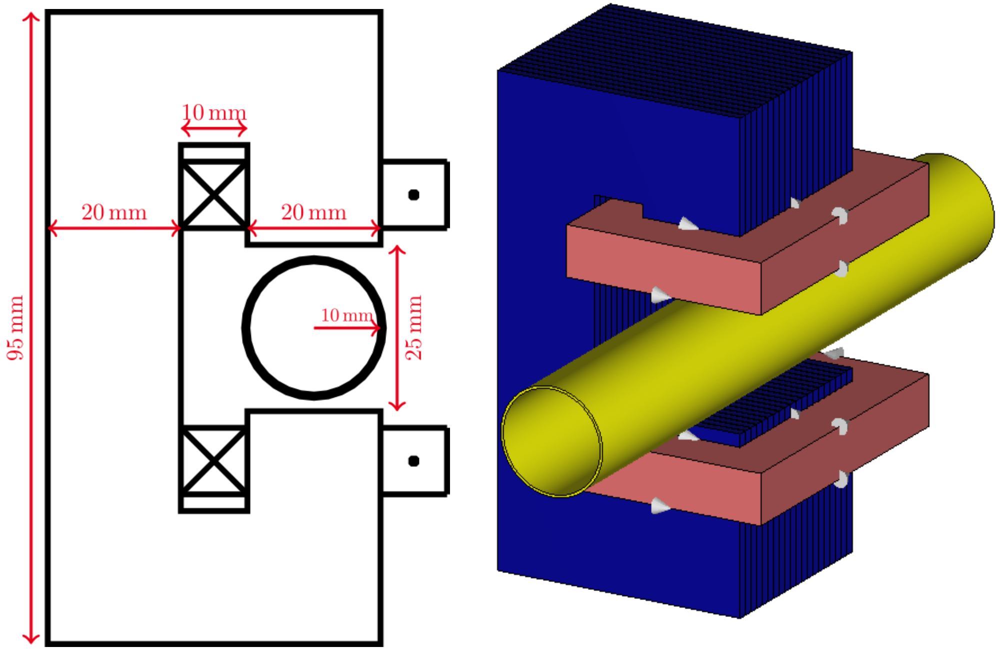
VERIFICATION

- **Toy model specifics:**
  - Iron yoke**:** length = 40 mm, lamination thickness = 1.83 mm
  - Copper beam pipe**:** thickness = 0.5 mm, length = 140 mm
  - Coils: current **=** 10 A (peak), # turns = 250
  - Frequency domain simulation via CST Studio Suite®



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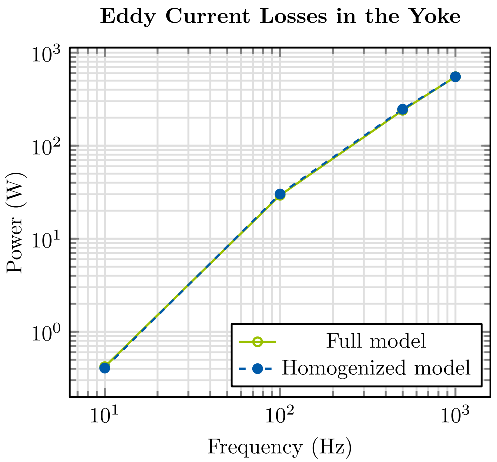
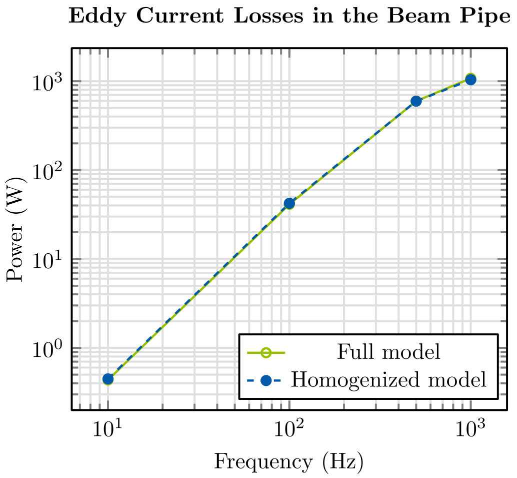
VERIFICATION

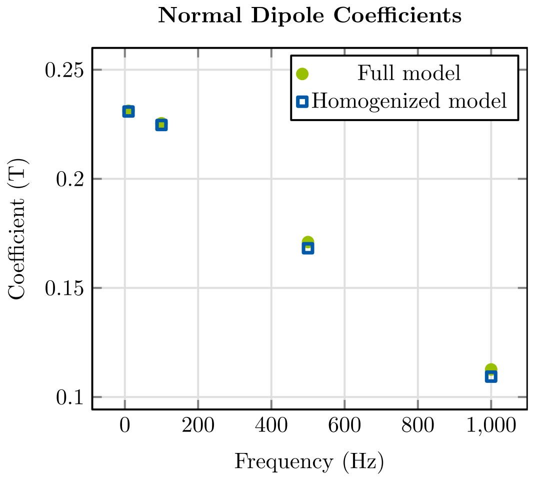
EDDY CURRENT LOSSES





- Eddy current losses are well approximated

MULTIPOLE COEFFICIENTS



Multipole coeff.	Average rel. error
Dipole	1 %
Quadrupole	5 %
Sextupole	2 %

- Aperture field is well approximated

- Note that the simulation time is reduced from several hours to just a few minutes !

➔ After comparing to other techniques, we decided to **use this technique to simulate the corrector magnets**

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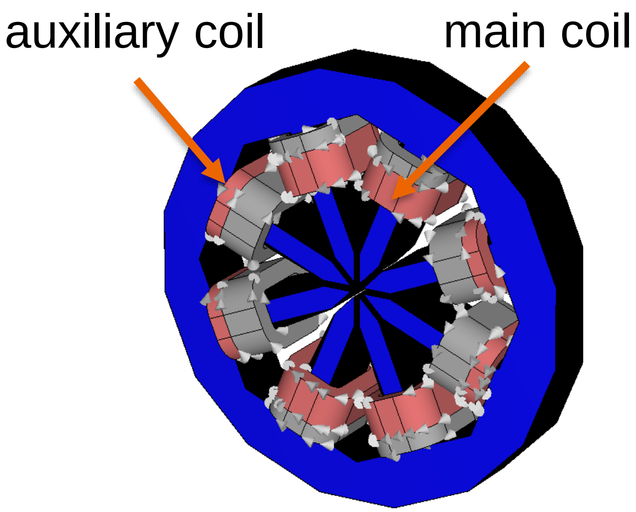
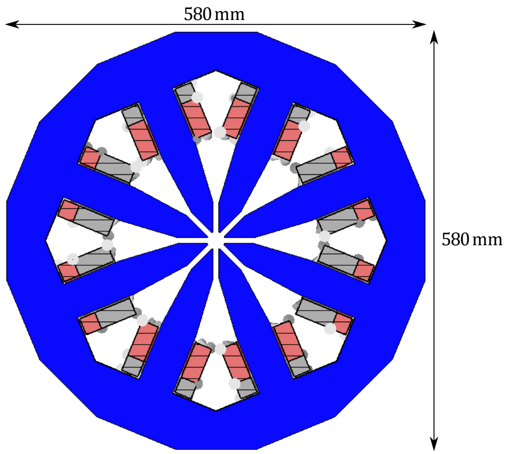
MODEL DESCRIPTION

- **Dipole** corrector with **octupole-like design**
- **Coils**:
  - 4 main coils: current = 15 A (peak), # turns = 65
  - 4 auxiliary coils: current = 15 A (peak), # turns = 27
- **Iron yoke:**
  - Diameter = 580 mm, length = 90 mm
  - Lamination thickness = 0.5 mm





Design by A. Aloev (DESY), inspired by APS

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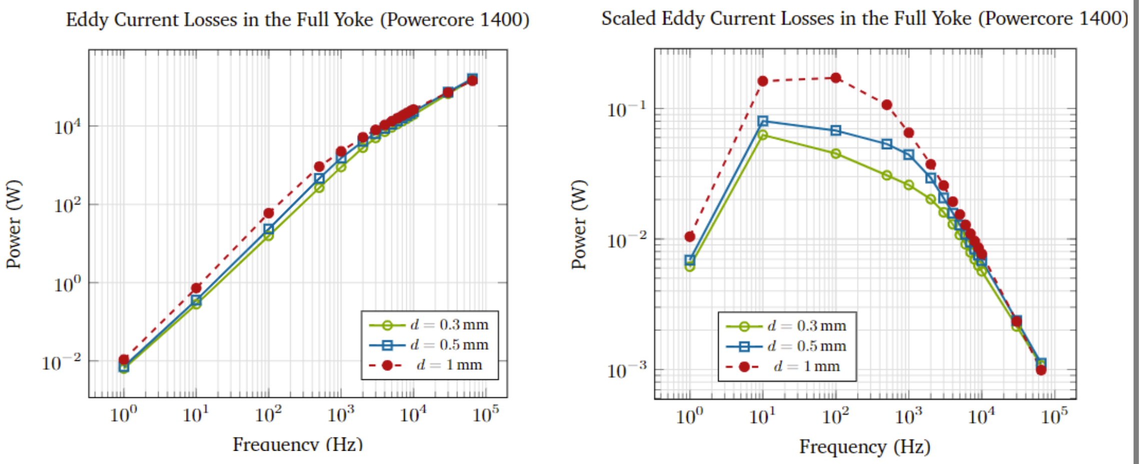
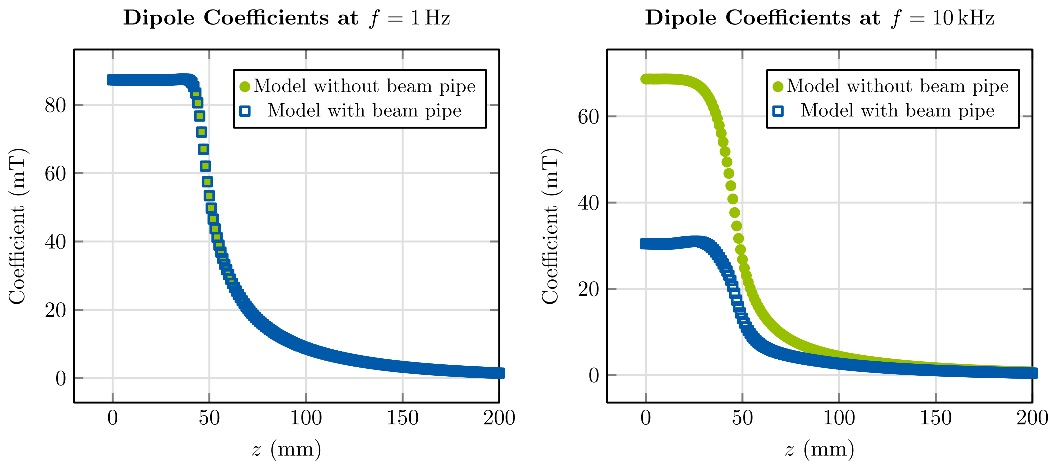
OVERVIEW OF STUDIES

EDDY CURRENT LOSSES



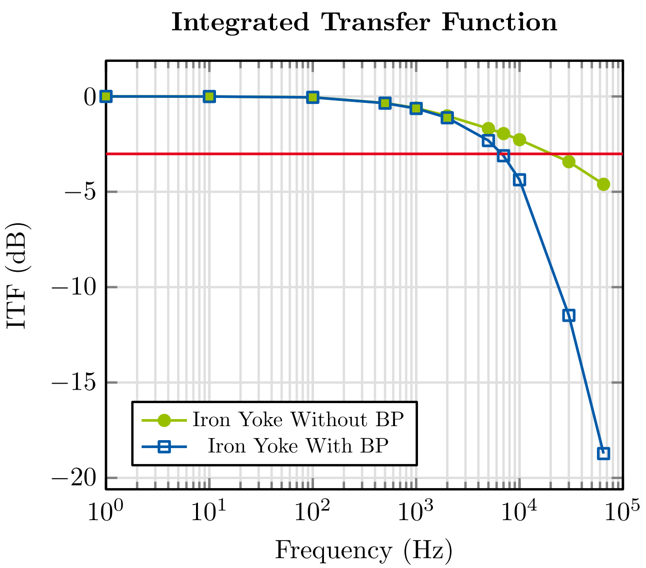
- At lower frequencies: $P_{\text{eddy}}\ \alpha\ f^2$, as expected analytically
- Lamination thickness only important at frequencies ≤ 1 kHz

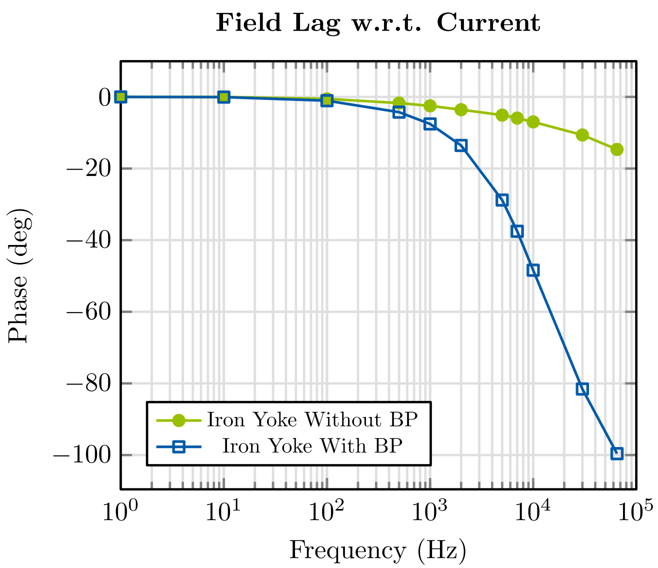
MULTIPOLE COEFFICIENTS



- Dipole field is attenuated due to eddy currents
- Beam pipe causes much stronger attenuation at higher frequencies and slight increase in effective length

OVERVIEW OF STUDIES (CONT.)





$$\mathrm{ITF}(f) = \frac{\int_l B_1(z,f)\,\mathrm{d}z}{\int_l B_1(z,f = 1\mathrm{Hz})\,\mathrm{d}z}$$

Yoke material	3 dB bandwidth	Phase shift at bandwidth
Iron	7 kHz	38°
M-19 Steel	10 kHz	46°
1010 Steel	7 kHz	38°

- Integrated transfer function and field lag (phase difference between current and aperture field) are of high interest for design of feedback control
- We compute both from our simulations for different yoke materials, different lamination thicknesses, and other design options

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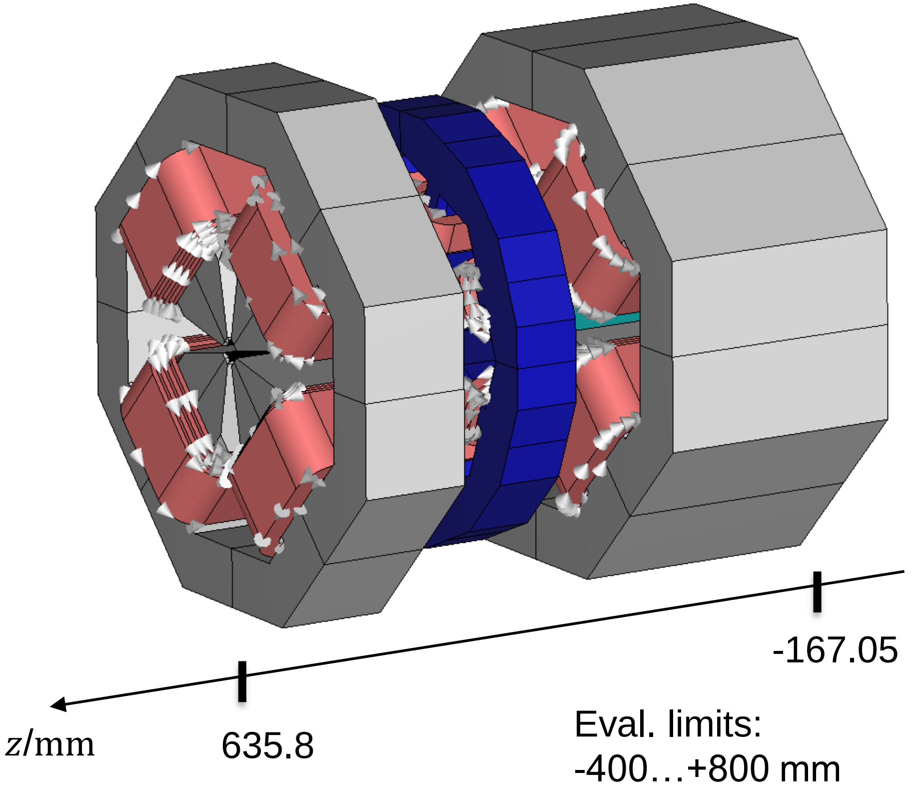
MODEL DESCRIPTION

- Corrector magnet with two neighboring quadrupole magnets
- AC currents in corrector coils, DC currents in quadrupole coils
- Quadrupole yokes are solid, corrector yoke is laminated
- Distance between corrector yoke and quadrupole yokes ~ 11.5 cm

➔ Cross talk must be investigated



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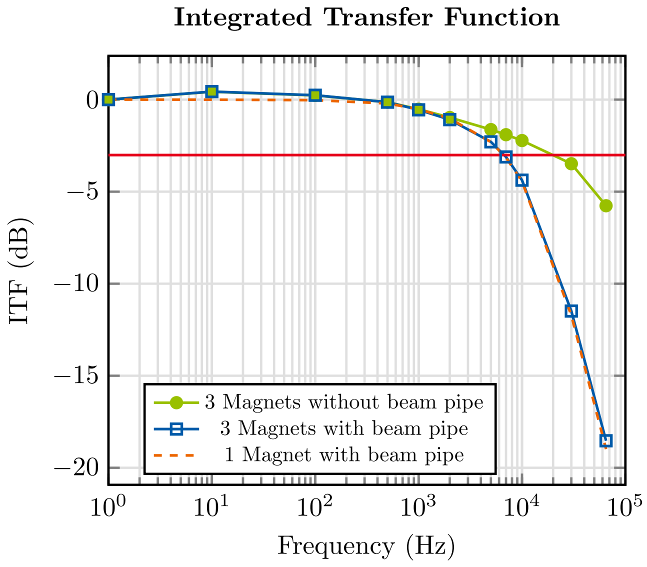
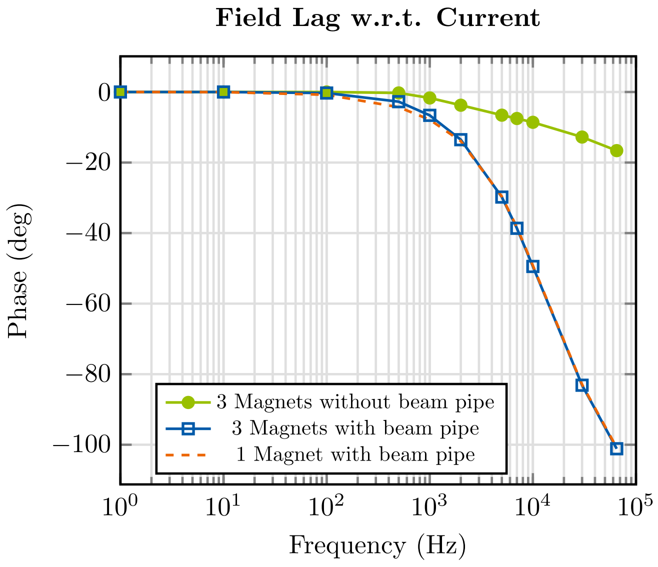
OVERVIEW OF STUDIES





	Model without beam pipe	Model with beam pipe
3 dB bandwidth	20 kHz	7 kHz
Phase shift at bandwidth	11°	39°

- Very similar results as for the model without neighboring quadrupoles
- **Main difference**: at low frequencies, a ~0.7 dB peak is occurring in the ITF of the model with the neighboring quadrupoles

➔ Potential problem for feedback control

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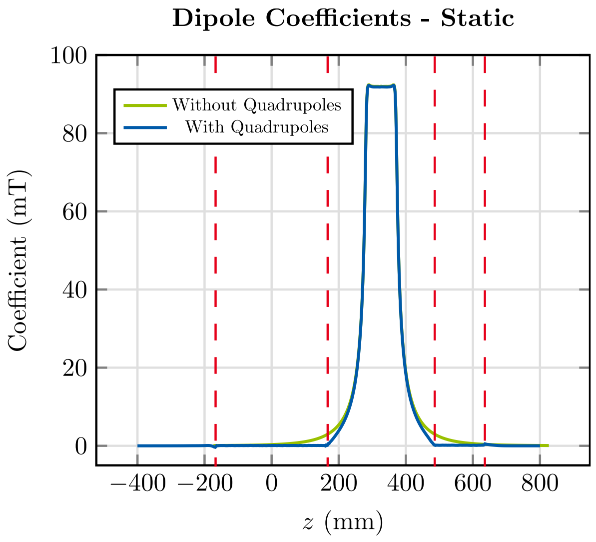
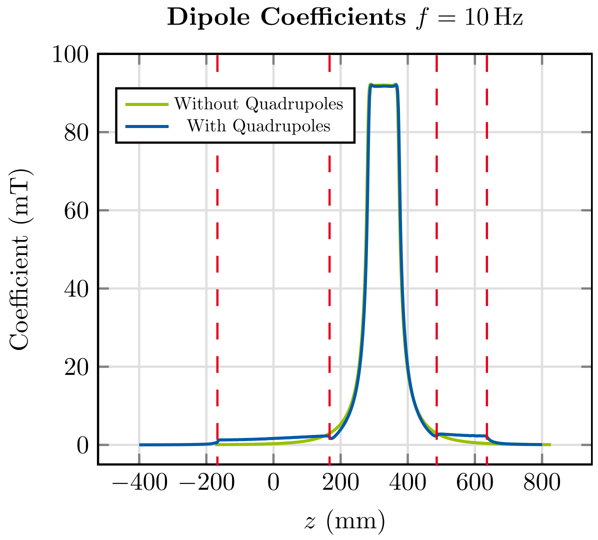
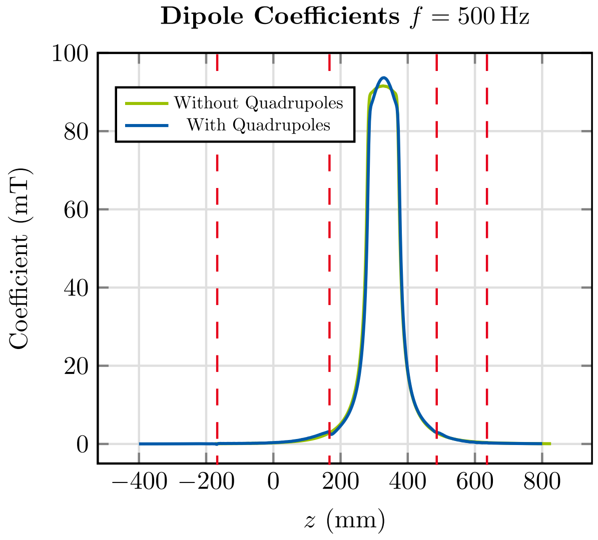
OVERVIEW OF STUDIES (CONT.)







- **At low frequencies** ($f \leq 100$ Hz), we observe a **parasitic dipole** component **inside the quadrupole magnets**
- This dipole component is due to eddy currents induced in the quadrupole yokes by the AC corrector field

➔ Peak in ITF at low frequencies

➔ Shift of the center of mass (~ 0.5 cm at most)

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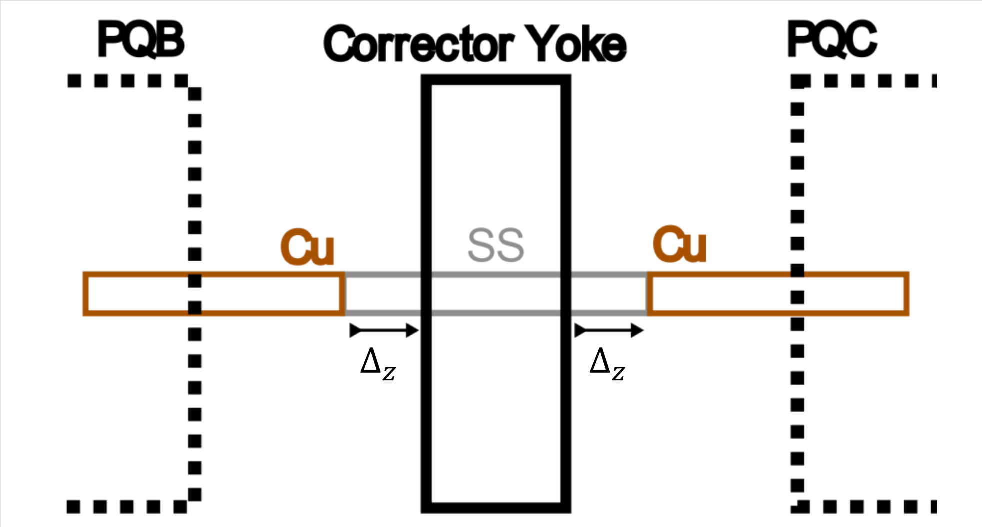
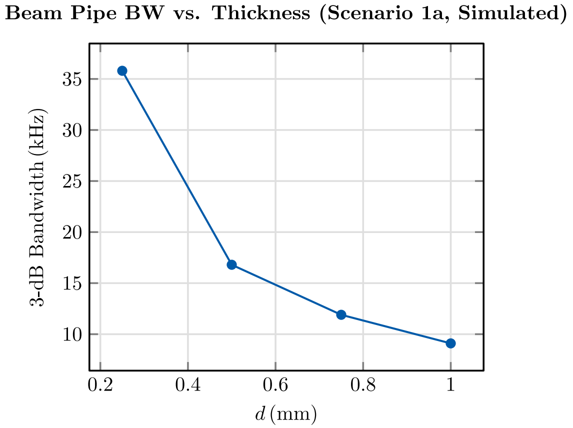
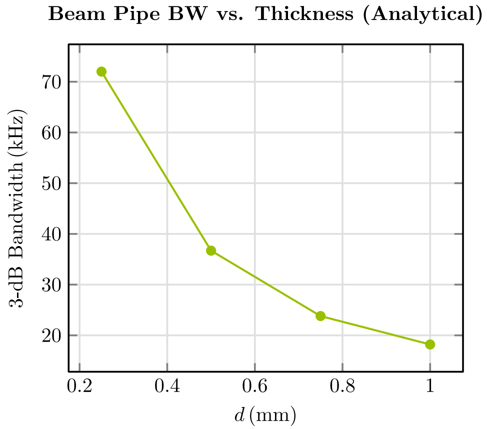
OVERVIEW OF STUDIES (CONT.)

Investigate different scenarios characterized by **distance $\boldsymbol{\Delta_z}$** from corrector yoke to copper parts of beam pipe for different beam pipe **thicknesses $\boldsymbol{d}$**



d	Δ_z = 0 mm (1c1)		Δ_z = 18 mm (1a2)		Δ_z = 29 mm (1a)	
	3dB-BW	Phase Shift at BW	3dB-BW	Phase Shift at BW	3dB-BW	Phase Shift at BW
1 mm	4.5 kHz	−21.3°	8.1 kHz	−34.7°	9.1 kHz	−37.9°
0.5 mm	9.5 kHz	−23.0°	15.2 kHz	−31.5°	16.8 kHz	−33.7°





➔ Scaling of BW with thickness as predicted by ana. formula, but ana. formula does not take material transition into account

B. Podobedov et al., "Eddy Current Shielding by Electrically Thick Vacuum Chambers" (2009)

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THEORY

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- To incorporate non-linear BH-curves into simulations: combine homogenization technique and harmonic balance FEM (HBFEM)
- HBFEM is a technique to approximate periodic solutions of nonlinear transient PDEs in frequency domain
- **Example**: excitation current with 1st and 3rd harmonic, include field quantities up to 3rd harmonic

$$\nabla \times (\nu(t)\nabla \times \vec{A}(t)) + \sigma \frac{\partial \vec{A}(t)}{\partial t} = \vec{J}_s(t)$$

$$\nabla \times (\underline{\nu}(\omega) \circledast \nabla \times \underline{\vec{A}}(\omega)) + j\omega\sigma\underline{\vec{A}}(\omega) = \underline{\vec{J}}_s(\omega)$$

+Homogenization →

$$\begin{bmatrix} K_{\underline{\overline{\overline{\nu}}}_0(3\omega_f)} + 3jw_f M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 & 0 \\ K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_f)} + jw_f M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} & 0 \\ 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(\omega_f)} - jw_f M_{\overline{\overline{\sigma}}} & K_{\underline{\nu}_2} \\ 0 & 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\overline{\overline{\nu}}}_0(3\omega_f)} - 3jw_f M_{\overline{\overline{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 \\ \underline{j}_1 \\ \underline{j}_{-1} \\ \underline{j}_{-3} \end{bmatrix}$$

S. Yamada and K. Bessho (1988)
H. De Gersem, H. Vande Sande, K. Hameyer (2001)

THEORY

- To resolve the nonlinearity: bring off-diagonal terms to the right-hand side
- Iterate until energy does not change anymore

$$\begin{bmatrix} K_{\underline{\underline{\nu}}_0(3\omega_f)} + 3jw_f M_{\underline{\underline{\sigma}}} & K_{\underline{\nu}_2} & 0 & 0 \\ K_{\underline{\nu}_{-2}} & K_{\underline{\underline{\nu}}_0(\omega_f)} + jw_f M_{\underline{\underline{\sigma}}} & K_{\underline{\nu}_2} & 0 \\ 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\underline{\nu}}_0(\omega_f)} - jw_f M_{\underline{\underline{\sigma}}} & K_{\underline{\nu}_2} \\ 0 & 0 & K_{\underline{\nu}_{-2}} & K_{\underline{\underline{\nu}}_0(3\omega_f)} - 3jw_f M_{\underline{\underline{\sigma}}} \end{bmatrix} \underbrace{\begin{bmatrix} \underline{a}_3 \\ \underline{a}_1 \\ \underline{a}_{-1} \\ \underline{a}_{-3} \end{bmatrix}}_{\underline{\vec{a}}} = \begin{bmatrix} \underline{j}_3 \\ \underline{j}_1 \\ \underline{j}_{-1} \\ \underline{j}_{-3} \end{bmatrix}$$

Initialize $K_{\underline{\underline{\nu}}_0(3\omega_f)}, K_{\underline{\underline{\nu}}_0(\omega_f)}, K_{\underline{\nu}_2}, K_{\underline{\nu}_{-2}}$ and solve for $\underline{\vec{a}}^{(1)}$

For $i = 1, 2, 3...$ **until convergence**

Compute $\underline{\vec{b}}^{(i)} = \nabla \times \underline{\vec{a}}^{(i)}$

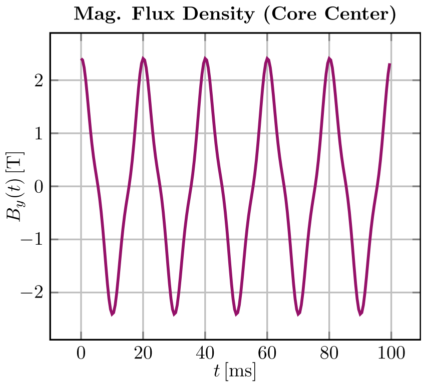
Transform in time domain $\underline{\vec{b}}^{(i)} \rightarrow \vec{B}(t)$

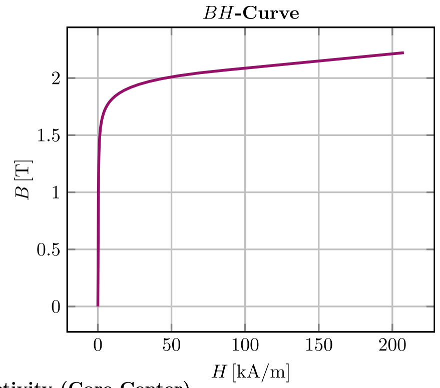
Insert $\|\vec{B}(t)\|$ in BH-curve to get $\|\vec{H}(t)\|$ and compute $\nu(t)$

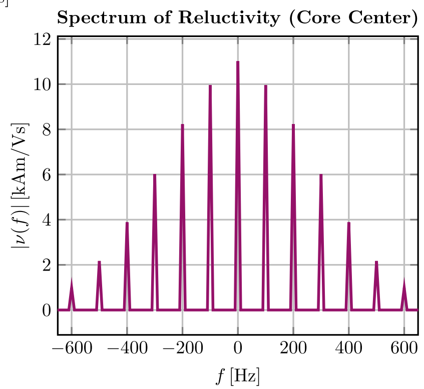
Fourier Transform $\nu(t) \rightarrow \underline{\nu}_0^{(i)}, \underline{\nu}_2^{(i)}, \underline{\nu}_{-2}^{(i)}$

Solve

$$\begin{bmatrix} K_{\underline{\underline{\nu}}_0^{(i)}(3\omega_f)} + 3j\omega_f M_{\underline{\underline{\sigma}}} & 0 & 0 & 0 \\ 0 & K_{\underline{\underline{\nu}}_0^{(i)}(\omega_f)} + j\omega_f M_{\underline{\underline{\sigma}}} & 0 & 0 \\ 0 & 0 & K_{\underline{\underline{\nu}}_0^{(i)}(\omega_f)} - j\omega_f M_{\underline{\underline{\sigma}}} & 0 \\ 0 & 0 & 0 & K_{\underline{\underline{\nu}}_0^{(i)}(3\omega_f)} - 3j\omega_f M_{\underline{\underline{\sigma}}} \end{bmatrix} \begin{bmatrix} \underline{a}_3^{(i+1)} \\ \underline{a}_1^{(i+1)} \\ \underline{a}_{-1}^{(i+1)} \\ \underline{a}_{-3}^{(i+1)} \end{bmatrix} = \begin{bmatrix} \underline{j}_3 - K_{\underline{\nu}_2^{(i)}} \underline{a}_1^{(i)} \\ \underline{j}_1 - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_3^{(i)} - K_{\underline{\nu}_2^{(i)}} \underline{a}_{-1}^{(i)} \\ \underline{j}_{-1} - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_1^{(i)} - K_{\underline{\nu}_2^{(i)}} \underline{a}_{-3}^{(i)} \\ \underline{j}_{-3} - K_{\underline{\nu}_{-2}^{(i)}} \underline{a}_{-1}^{(i)} \end{bmatrix}$$







S. Yamada and K. Bessho (1988)
H. De Gersem, H. Vande Sande, K. Hameyer (2001)

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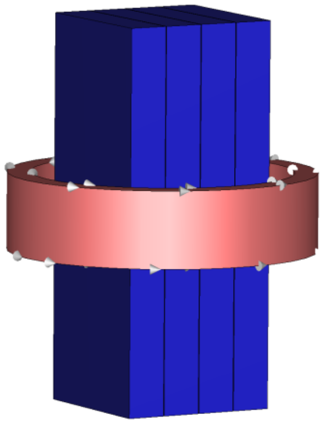
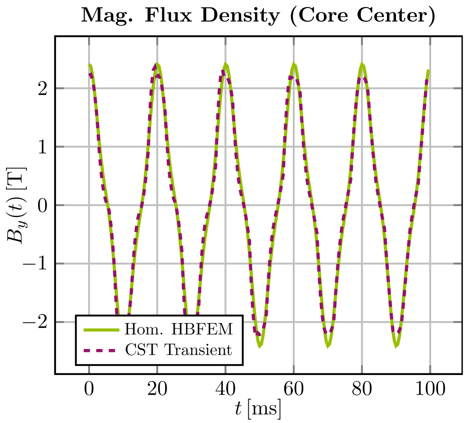
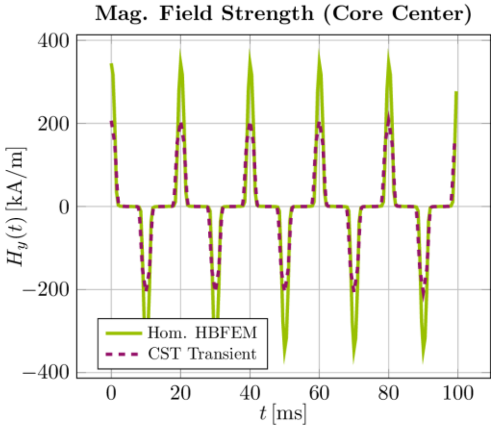
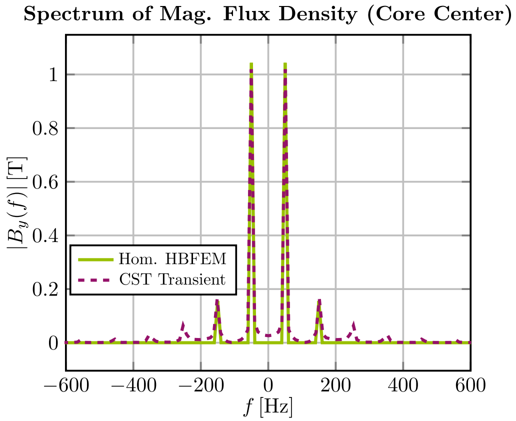
VERIFICATION

- Simple inductor with laminated core, excitation current: $I_s(t) = 1.5\text{ kA}\cos(2\pi 50\text{Hz}\,t) + 0.24\text{ kA}\cos(2\pi 150\text{Hz}\,t)$
- Compare results of HBFEM + homogenization (GetDP + Python) to transient CST simulation with individually resolved laminations









- Good agreement in magnetic flux density
- Larger differences in magnetic field strength
- **Suspicion: differences in magnetic field strength are due to not having included enough harmonics**

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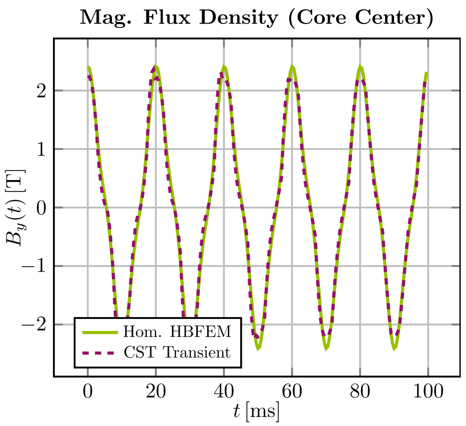
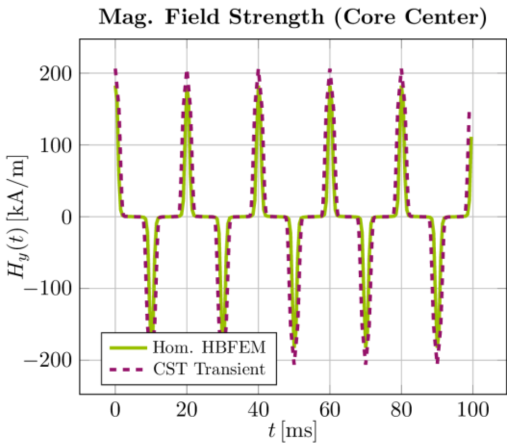
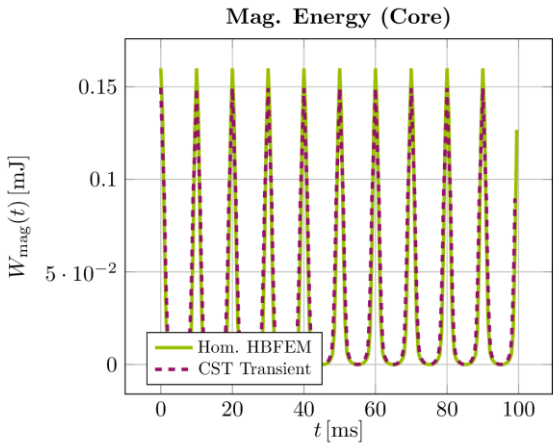
VERIFICATION (CONT.)

- After including the 5th harmonic in the analysis, we obtain:







- Still good agreement in magnetic flux density, large differences in magnetic field strength vanish
- Decent agreement in magnetic energy

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THEORY

- Current signal of corrector magnet: DC current + oscillations ➔ modify HBFEM method to include DC bias
- Again, we combine HBFEM with a homogenization technique

$$\nabla \times \left(\underline{\nu}(\omega) \circledast \nabla \times \vec{\underline{A}}(\omega) \right) + j\omega\sigma \vec{\underline{A}}(\omega) = \vec{\underline{J}}_s(\omega) \;\Rightarrow\; \nabla \times \left(\underline{\nu}_d(\omega) \circledast \nabla \times \vec{\underline{A}}(\omega) \right) + j\omega\sigma \vec{\underline{A}}(\omega) = \vec{\underline{J}}_s(\omega) - \nabla \times \vec{\underline{H}}_c(\omega)$$

chord reluctivity — differential reluctivity — magnetizing field strength

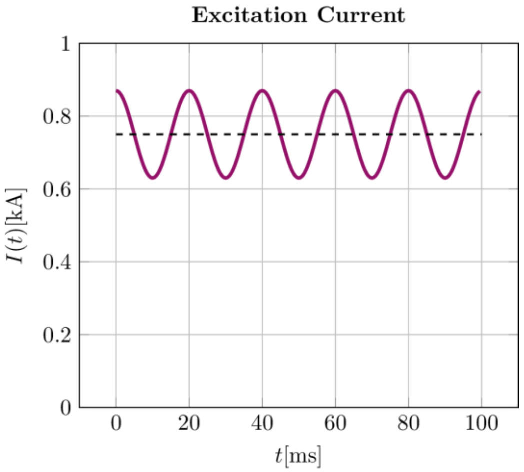
HOMOGENIZATION

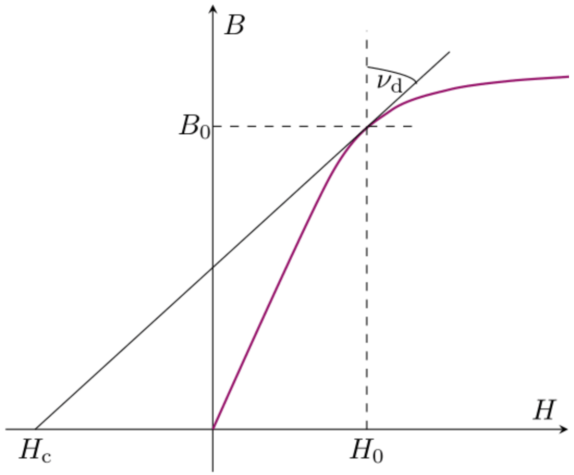
$$\nabla \times (\bar{\bar{\nu}} \nabla \times \vec{A}) + \nabla \times \left(\bar{\bar{\xi}} \nabla \times \frac{\partial \vec{A}}{\partial t} \right) + \bar{\bar{\sigma}} \frac{\partial \vec{A}}{\partial t} = \vec{J}_s$$

$$\bar{\bar{\nu}} = \frac{1}{\frac{\gamma}{\nu_{\text{Fe}}} + \frac{1-\gamma}{\nu_{\text{Iso}}}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (\nu_{\text{Fe}}\gamma + \nu_{\text{Iso}}(1-\gamma)) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{\xi}} = \frac{1}{12} \sigma_{\text{Fe}} d^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

J. Gyselinck et al., 1999





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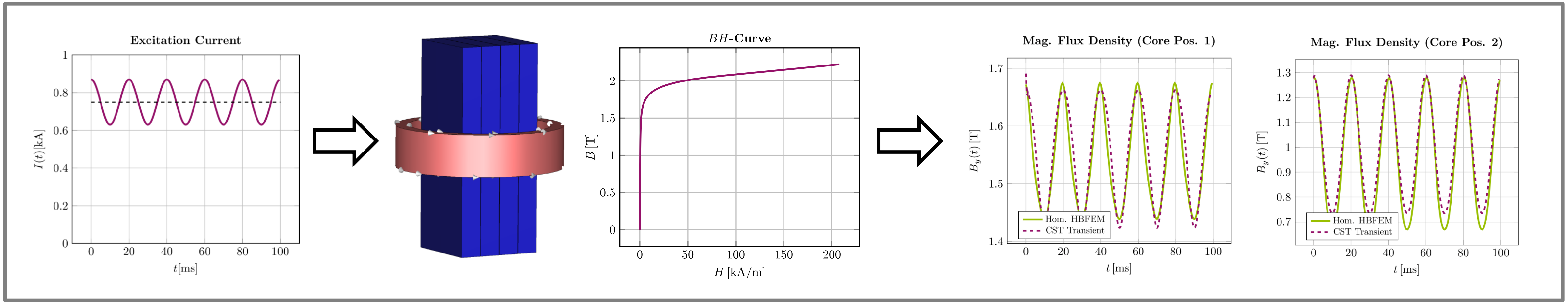
VERIFICATION

- Excitation current: $I_s(t) = 750A + 120\text{A}\cos(2\pi 50\text{Hz}\ t)$
- Compare again to transient CST simulation of toy model
- Results are promising, but agreement with transient simulation is worse than for method without DC bias

➔ Method must be further investigated and improved



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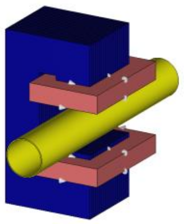
CONTENTS

CONCLUSION/OUTLOOK

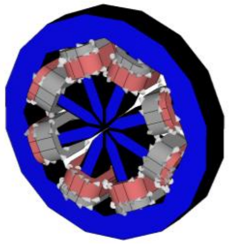
1. Verification of homogenization technique using **toy model**

➔ Good approximation of multipoles and power losses
➔ Simulation time reduced from several hours to a few min.



2. Application to **corrector magnet model**

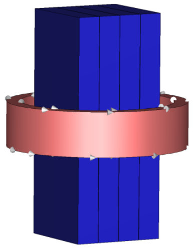
- Power losses, multipoles along axis
- Integrated transfer function and field lag
- Cross-talk with neighboring magnets

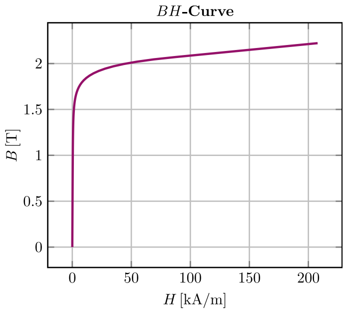


3. Treatment of **nonlinear material properties: homogenization + HBFEM**

- Good results for simple inductor model without DC bias
- Promising results for simple inductor model with DC bias, but needs improvement

➔ Improve method with DC bias and then move to practically more relevant model





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