# Assessment of Different Geometries for Single-Mode Cavities

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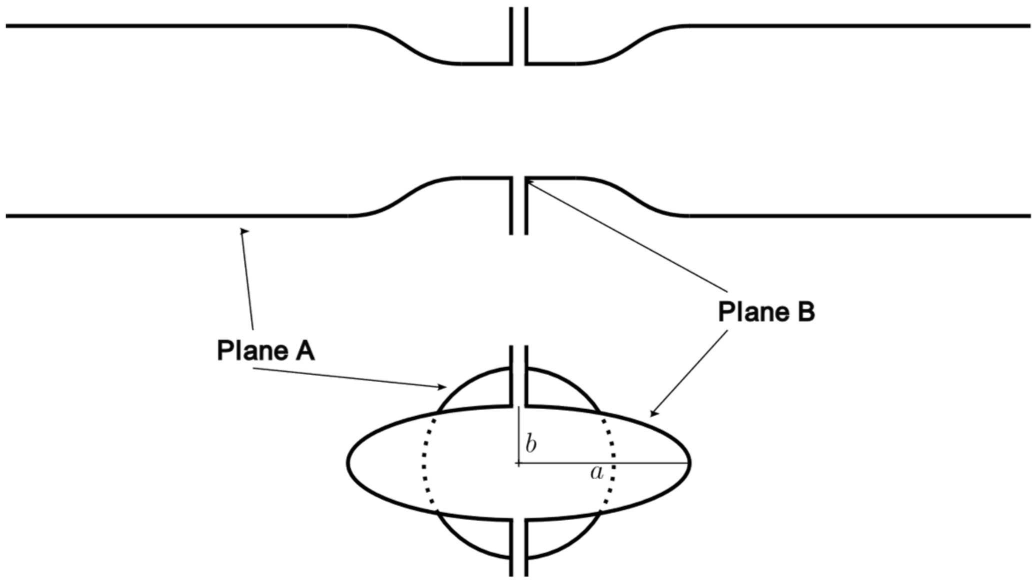
# Outline

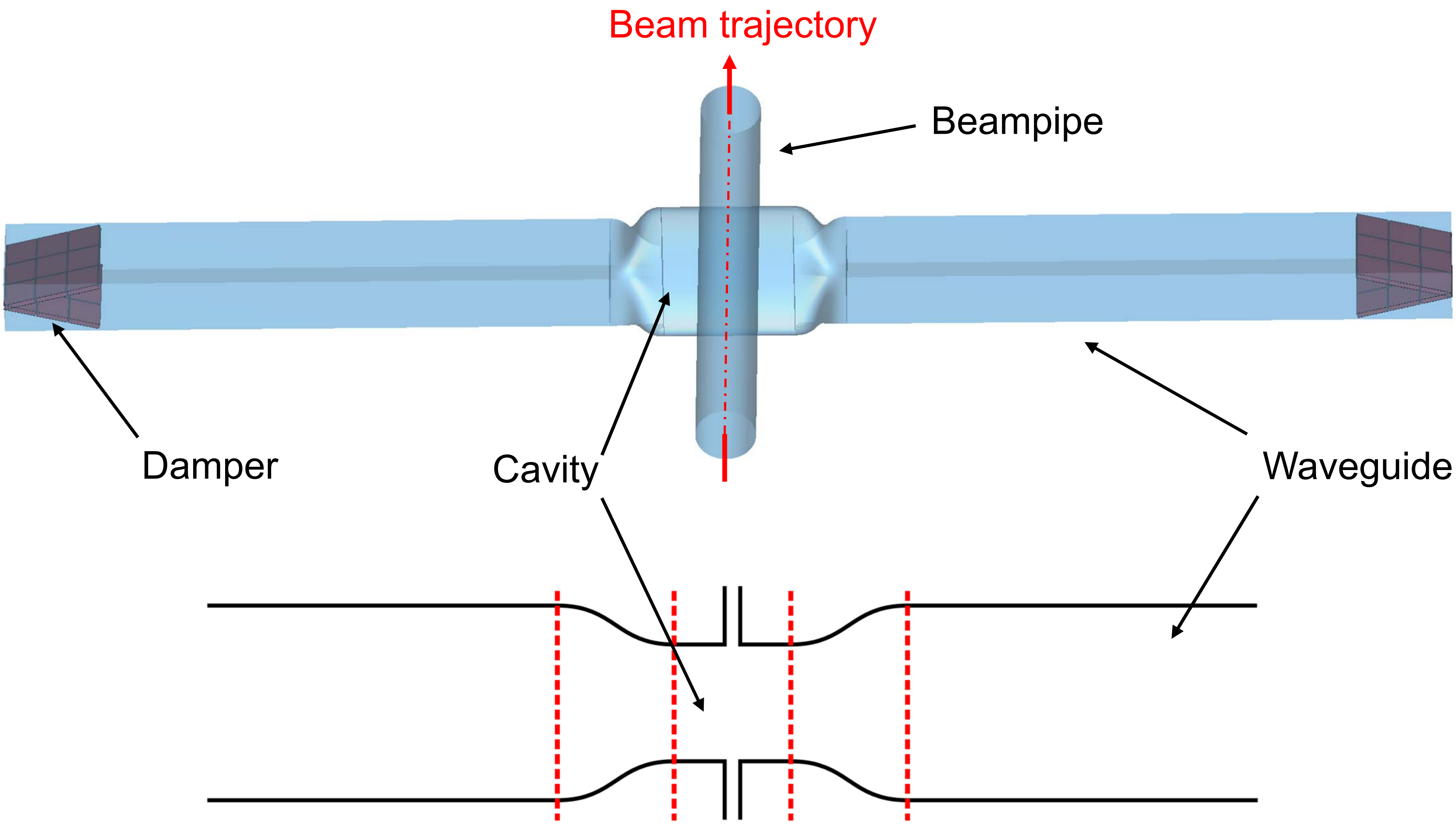
- Recap of Herminghaus's Cavity Proposal
- Design Goals
- Considered Geometries
- Naive Ansatz
- Practical Single-Mode-Cavity
- Conclusion and Outlook

# Herminghaus‘s Cavity Proposal

- Tapering a waveguide around the crossing beampipe
- Resonant frequency of ground mode is lowered beneath cutoff frequency of waveguide
- Higher order modes are free to travel down the waveguide to be damped



# Cavity Schematic



# The Downscaled BESSY-Cavity

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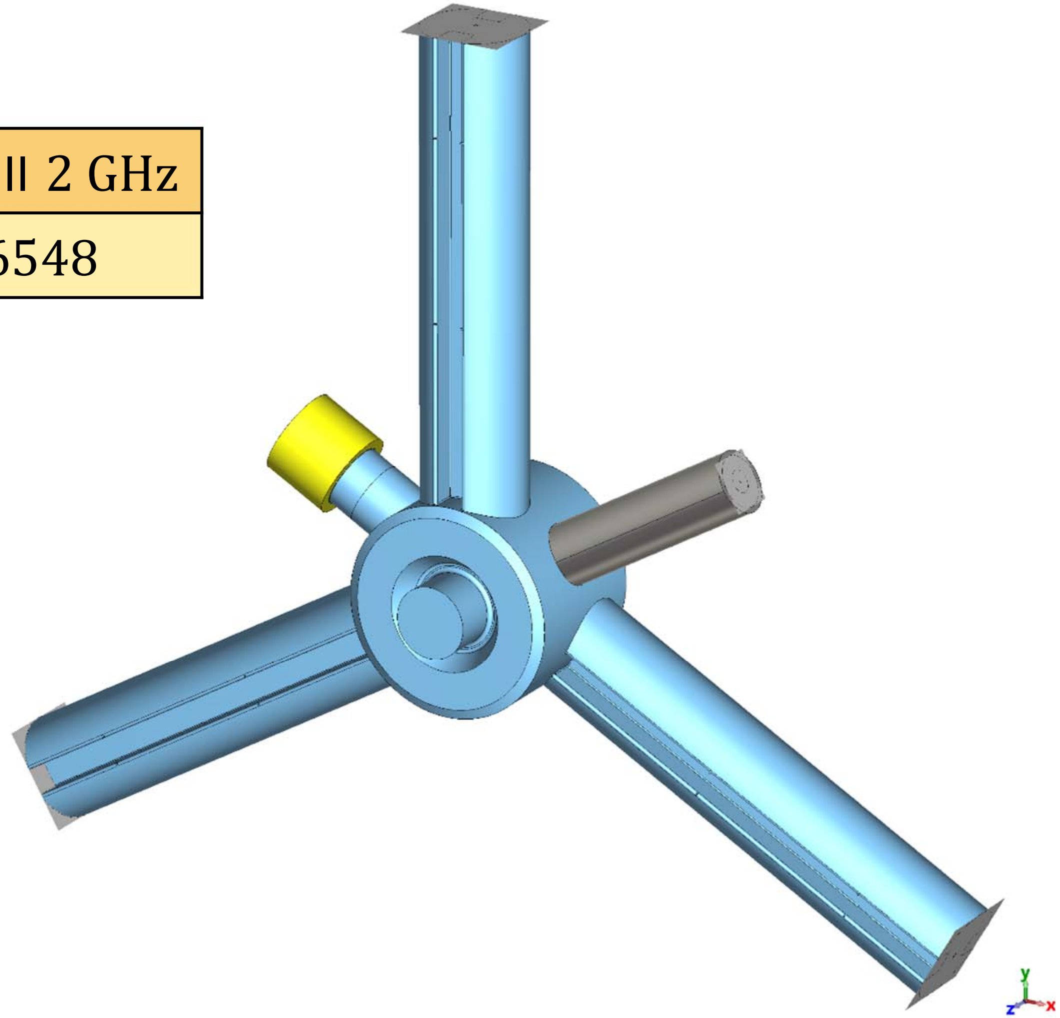
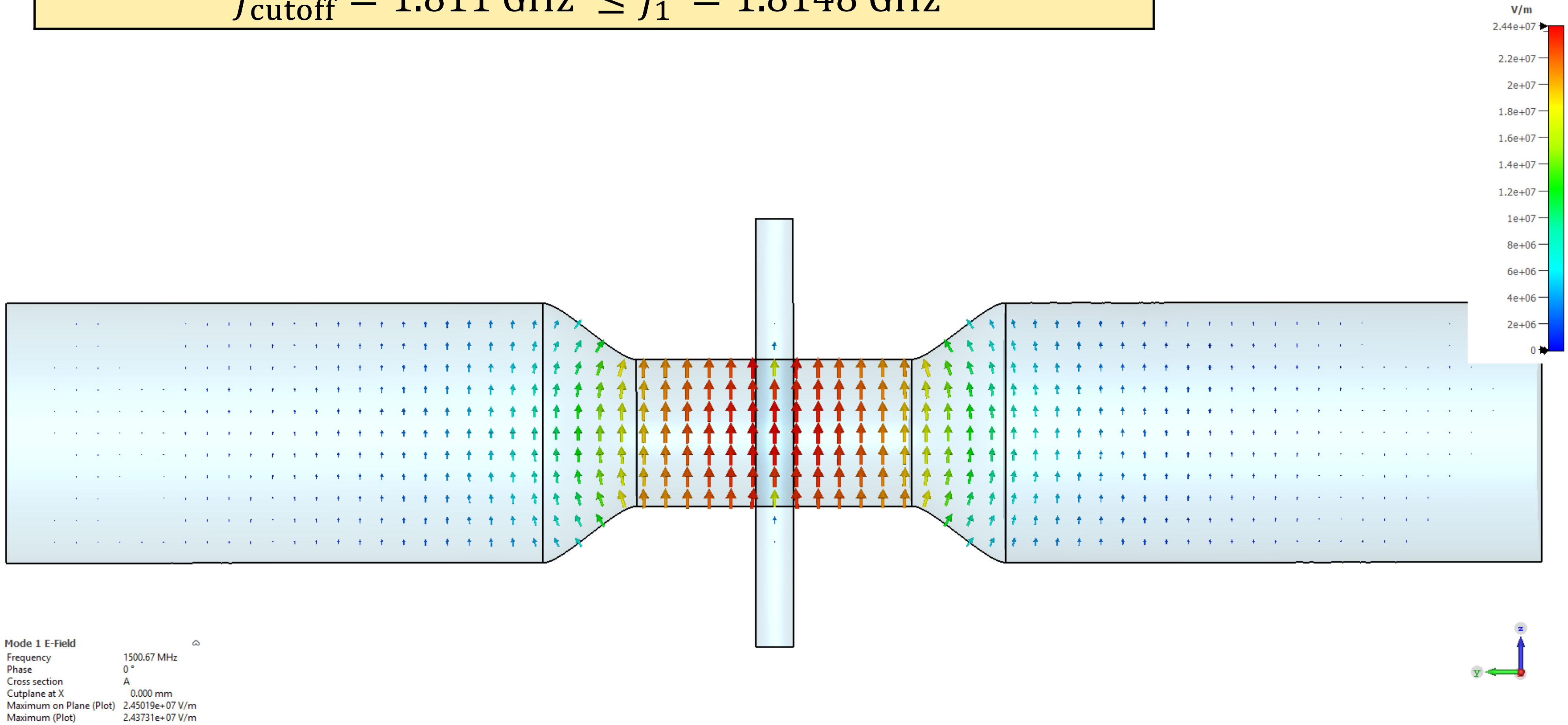
| | |
|---|---|
| $f_0 = 1.46$ GHz | 10 HOM's till 2 GHz |
| $Q_0 = 4642$ | $T_{\mathrm{Tr}} = 0.6548$ |



# Previous Simulation Results

| $f_0 = 1.5006\ \text{GHz}$ | $Q_0 = 17400$ | $T_{\text{Tr}} = 0.8704$ |
|---|---|---|
| $f_{\text{cutoff}} = 1.811\ \text{GHz} \leq f_1 = 1.8148\ \text{GHz}$ | | |



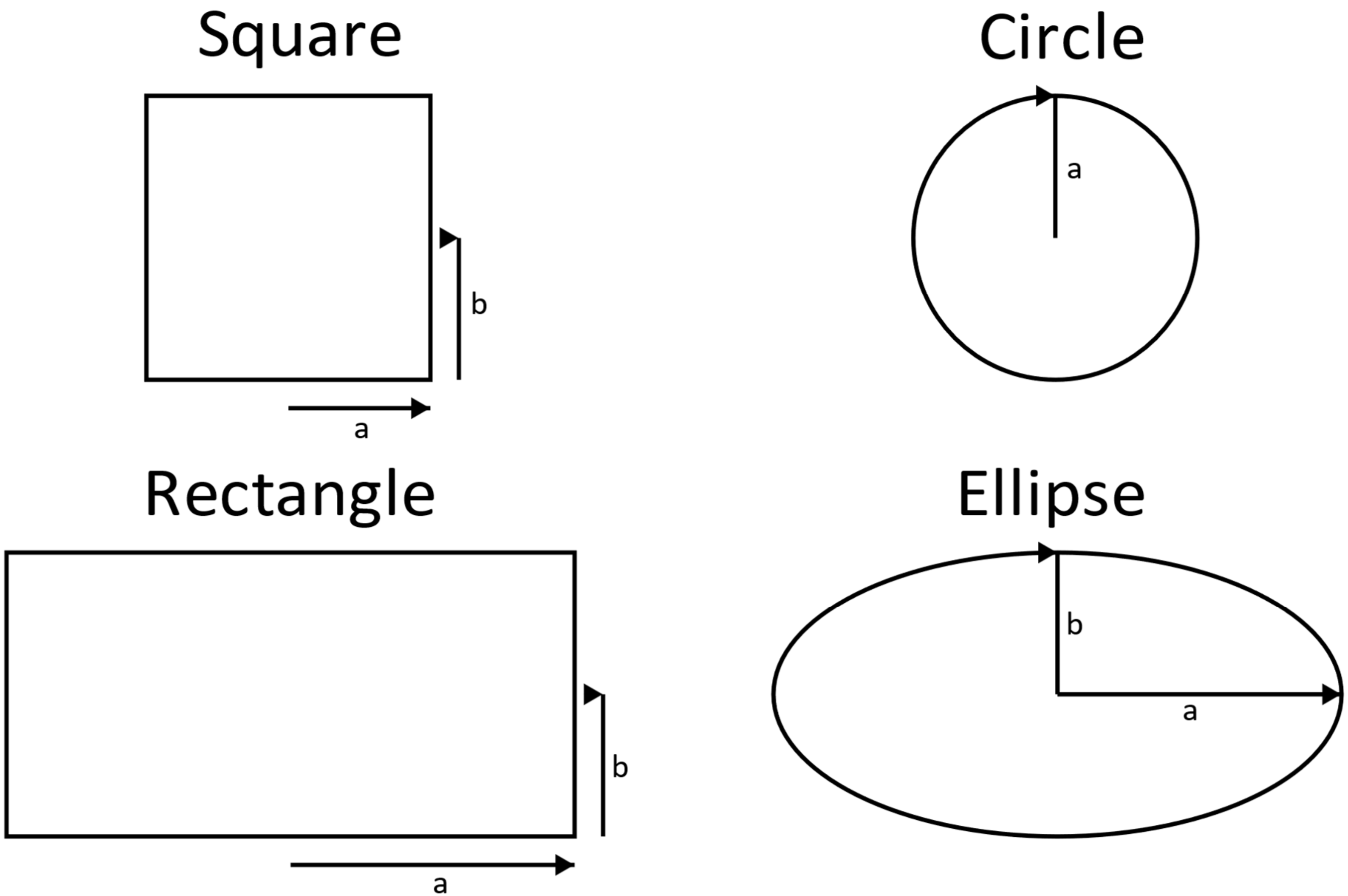
# Design Goals for the 3rd Harmonic Cavity

- The ground mode frequency: $f_0 = 1.5\,\mathrm{GHz}$
- Maximize the spacing to the 1st higher order mode
  - $\Delta f = f_1 - f_0$
- Maximize the Quality Factor $Q$
- Maximize the Transit Time Factor $T_{\mathrm{Tr}}$

$$Q_0 = \frac{\omega W}{P_{\mathrm{loss}}}, \qquad T_{\mathrm{Tr}} = \left| \frac{\int_{-L/2}^{L/2} E_0(s) \cos\left(\frac{\omega s}{c}\right) ds}{\int_{-L/2}^{L/2} E_0(s) ds} \right|$$

- Fixed beam pipe radius: $r_{\mathrm{bp}} = 23\,\mathrm{mm}$

# Considered Geometries



# General Geometric Dependency

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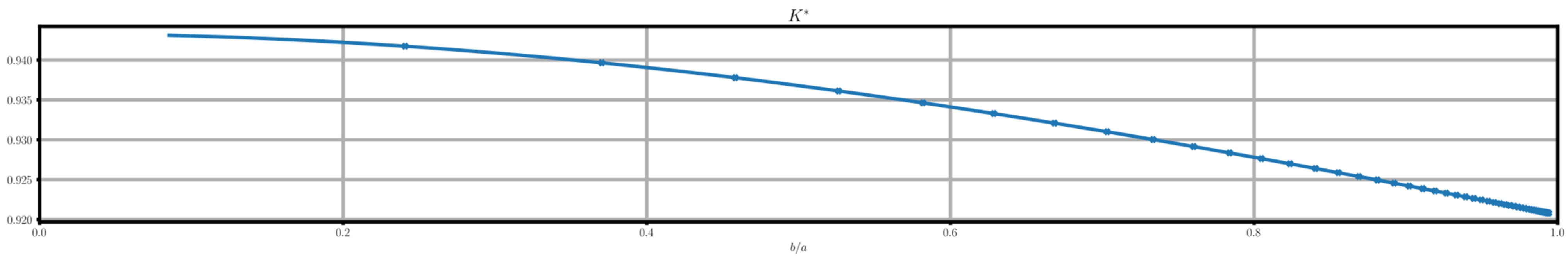
- Cutoff frequency of lowest waveguide mode for different cross-sectional geometries
  - Square and Rectangle: $f_{\mathrm{cutoff}} = \frac{c}{2} \cdot \frac{1}{a}$
  - Circle: $f_{\mathrm{cutoff}} = \frac{c}{2\pi} \cdot \frac{j'_{11}}{a}$,
    $j'_{11}$ – 1st root of the 1st derivative of the 1st order Bessel function
  - Ellipse: $f_{\mathrm{cutoff}} = \frac{c}{2} \cdot \frac{1}{a} \cdot K^*\left(\frac{b}{a}\right)$, $K^*\left(\frac{b}{a}\right) = \sqrt{\frac{q_{11}}{1-\left(\frac{b}{a}\right)^2}}$
    $q_{11}$ - 1st root of the 1st derivative of the 1st order odd modified Mathieu function



# General Geometric Dependency

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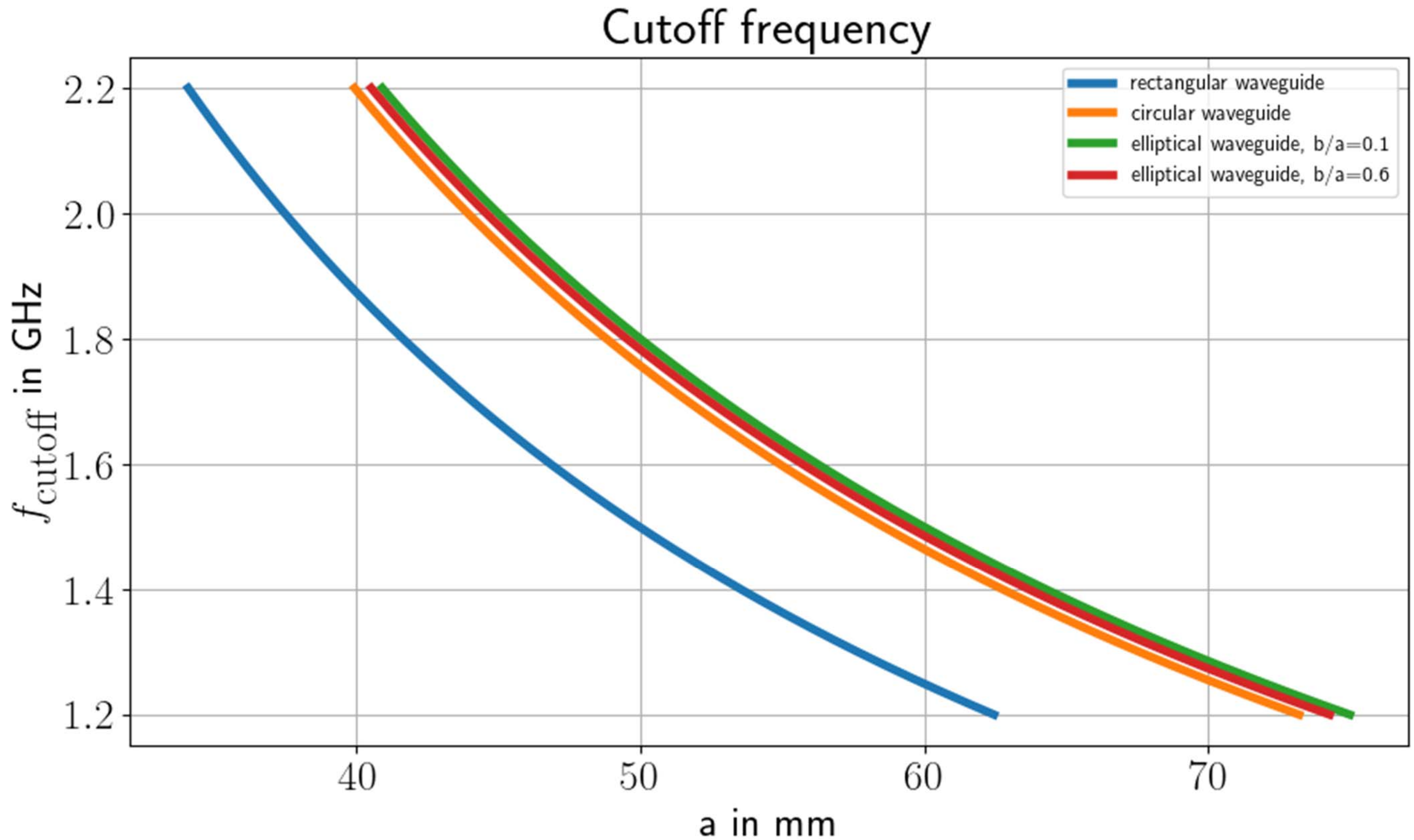
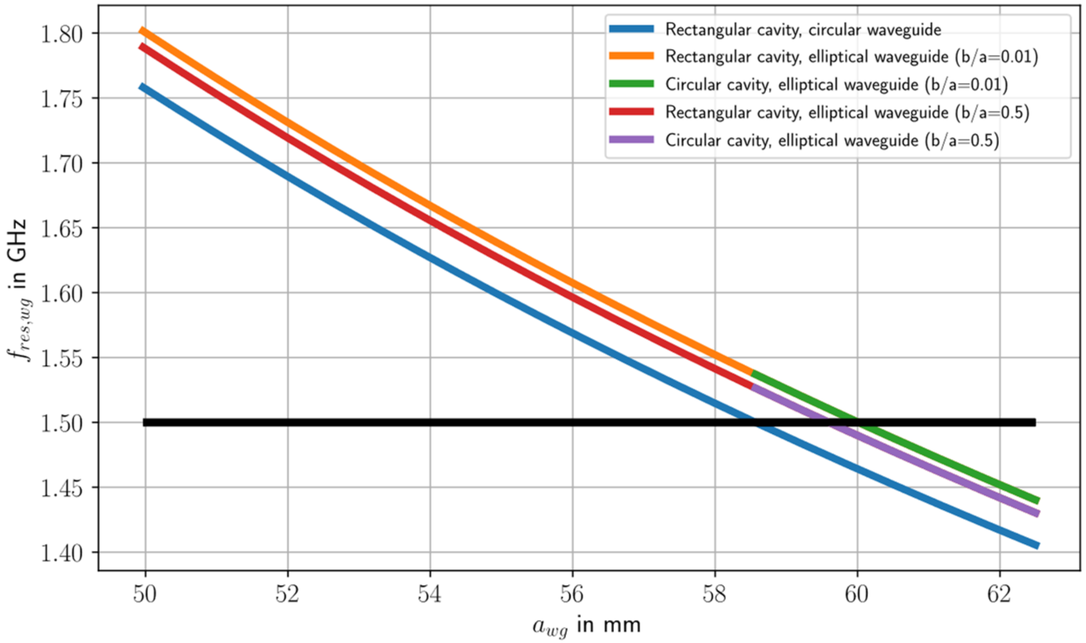


# Naive Ansatz

- Directly connecting the main cavity part to the waveguide
  - The waveguides cutoff frequency
    >
    resonant frequency of the cavity



# Naive Ansatz

- Rectangular cavity: $f_0 = 1.5\ \text{GHz}$

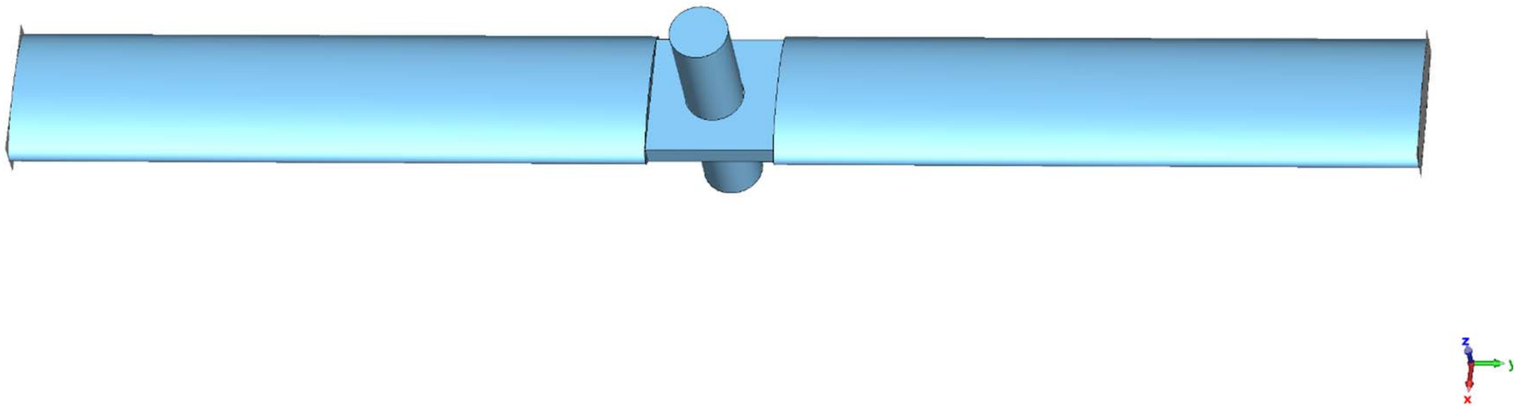
  $\Rightarrow a_{\text{cav}} = 49.965\ \text{mm},\ \ \text{b/a}_{\text{cav}} = 0.167$

- Elliptical Waveguide: $f_{\text{cutoff}} = 1.57\ \text{GHz}$

  $\Rightarrow a_{\text{wg}} = 49.965\ \text{mm},\ \ \text{b/a}_{\text{wg}} = 0.3$

- Very restricted geometry results in

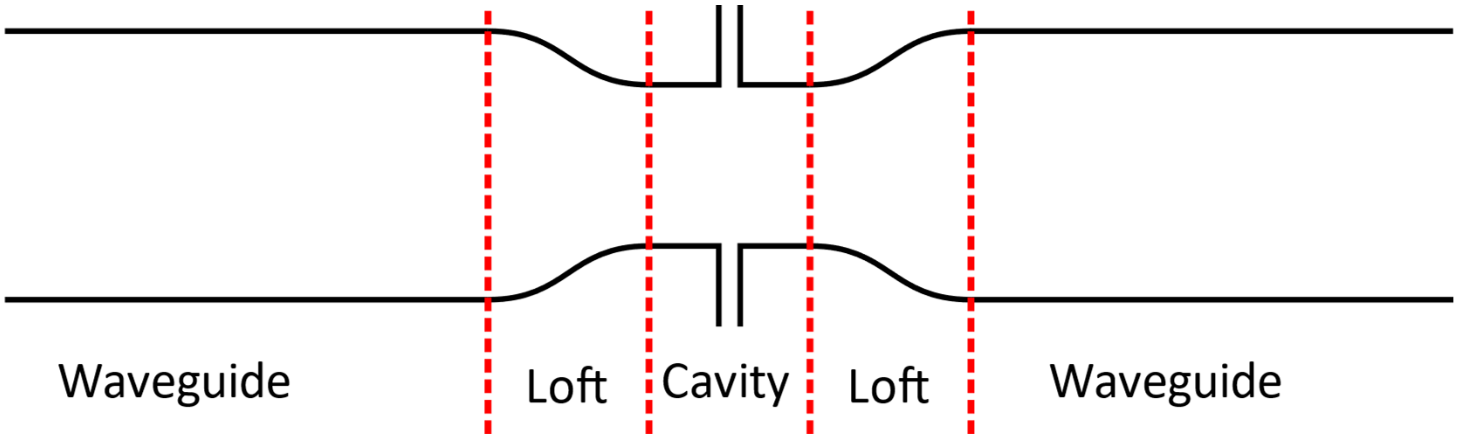
  $\Rightarrow Q_0 = 10200,\ T_{\text{Tr}} = 0.845$



# Improving the Ansatz

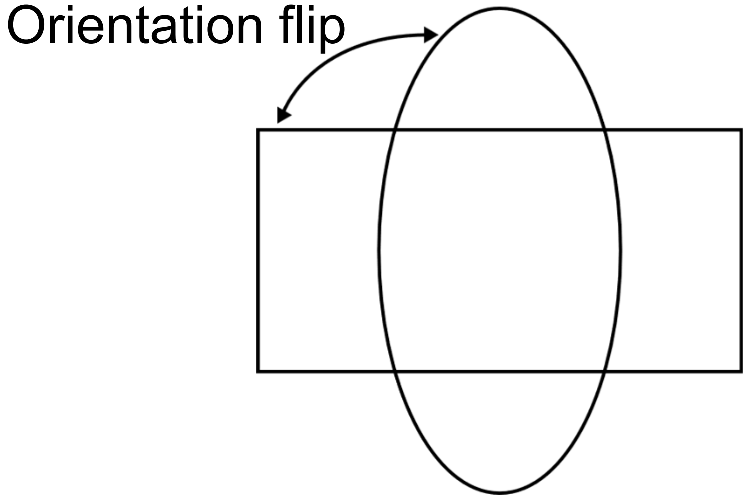
- The 2$^{nd}$ resonant frequency can be set above cutoff of the waveguide by using a loft

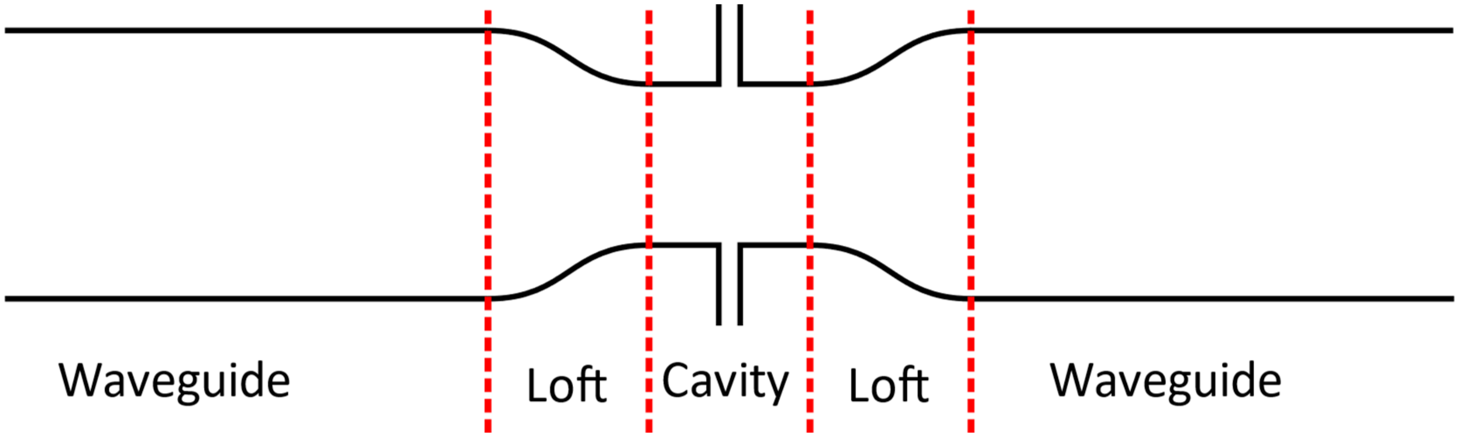
- The 1$^{st}$ resonant frequency is typically below the cutoff of a waveguide with the same cross-section
  - It has nothing to do with the behavior of a true cavity



# Simulation Results

- For all following results:
  - $f_0 = 1.5\ \mathrm{GHz}$
  - $f_{\mathrm{cutoff}} = 2\ \mathrm{GHz}$



# Simulation Results

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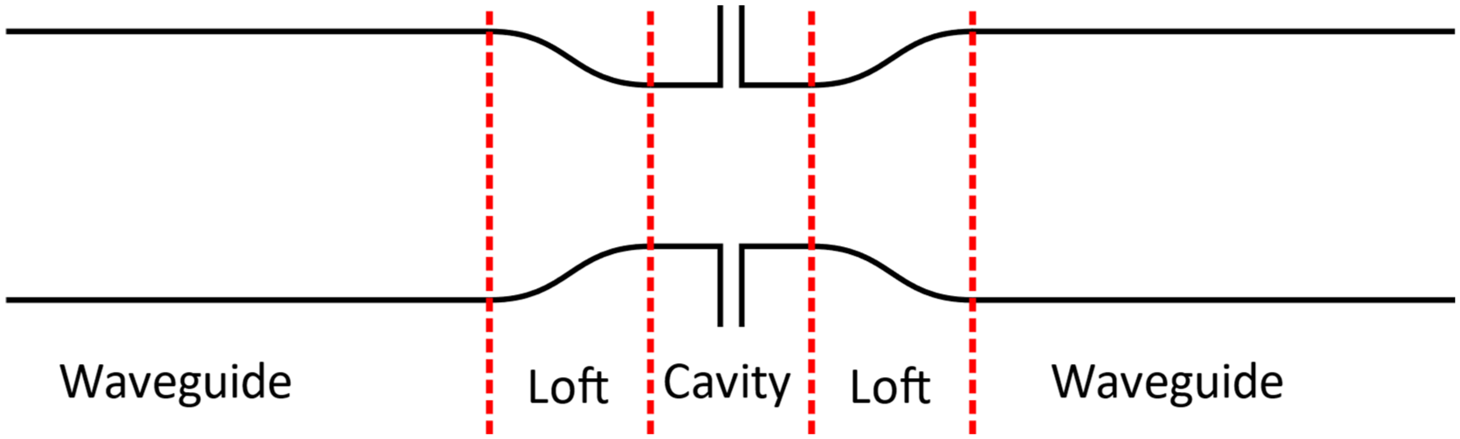
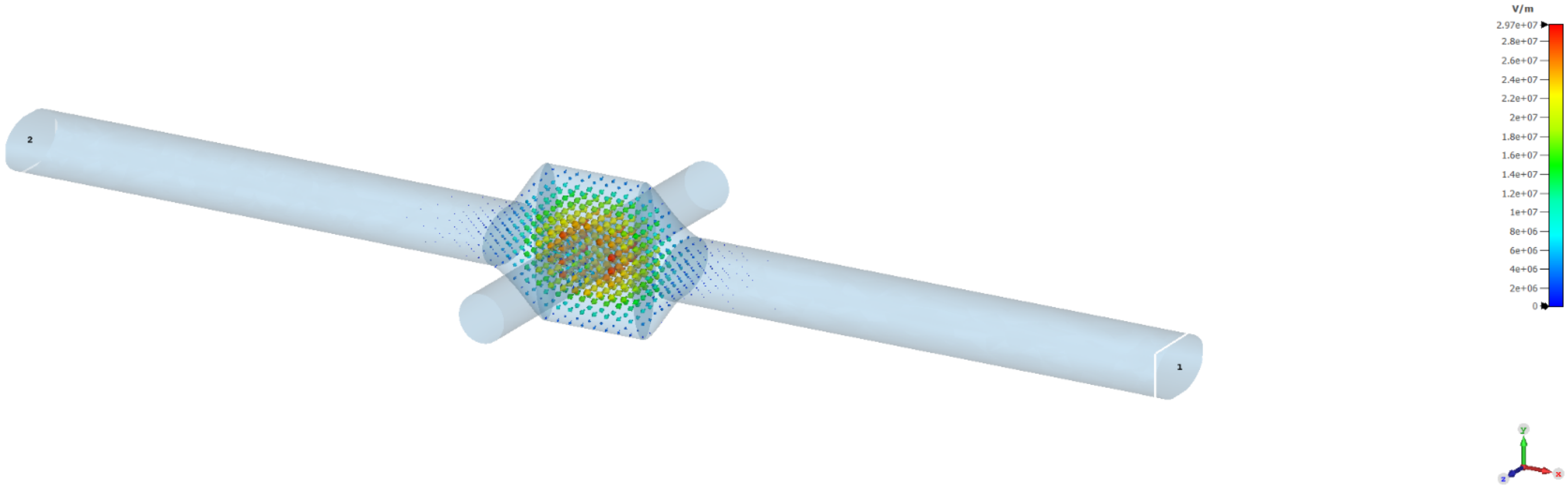
| | Square Wavguide | Flipped Rectangular Waveguide | Circular Waveguide | Flipped Elliptic Waveguide |
|---|---|---|---|---|
| **Rectangular Cavity** | $f_1 = 2.003$<br>$Q_0 = 14\,539$<br>$T_{\mathrm{TR}} = 0.808$ | $f_1 = 2.006$<br>$Q_0 = 13\,375$<br>$T_{\mathrm{TR}} = 0.818$ | $f_1 = 2.003$<br>$Q_0 = 16\,452$<br>$T_{\mathrm{TR}} = 0.808$ | $f_1 = 2.007$<br>$Q_0 = 14\,306$<br>$T_{\mathrm{TR}} = 0.816$ |
| **Elliptic Cavity** | $f_1 = 2.004$<br>$Q_0 = 17\,469$<br>$T_{\mathrm{TR}} = 0.754$ | $f_1 = 2.006$<br>$Q_0 = 15\,539$<br>$T_{\mathrm{TR}} = 0.789$ | $f_1 = 2.004$<br>$Q_0 = 17\,484$<br>$T_{\mathrm{TR}} = 0.784$ | $f_1 = 2.005$<br>$Q_0 = 15\,454$<br>$T_{\mathrm{TR}} = 0.802$ |
| | Cavity had to be short | | Cavity had to be short | |

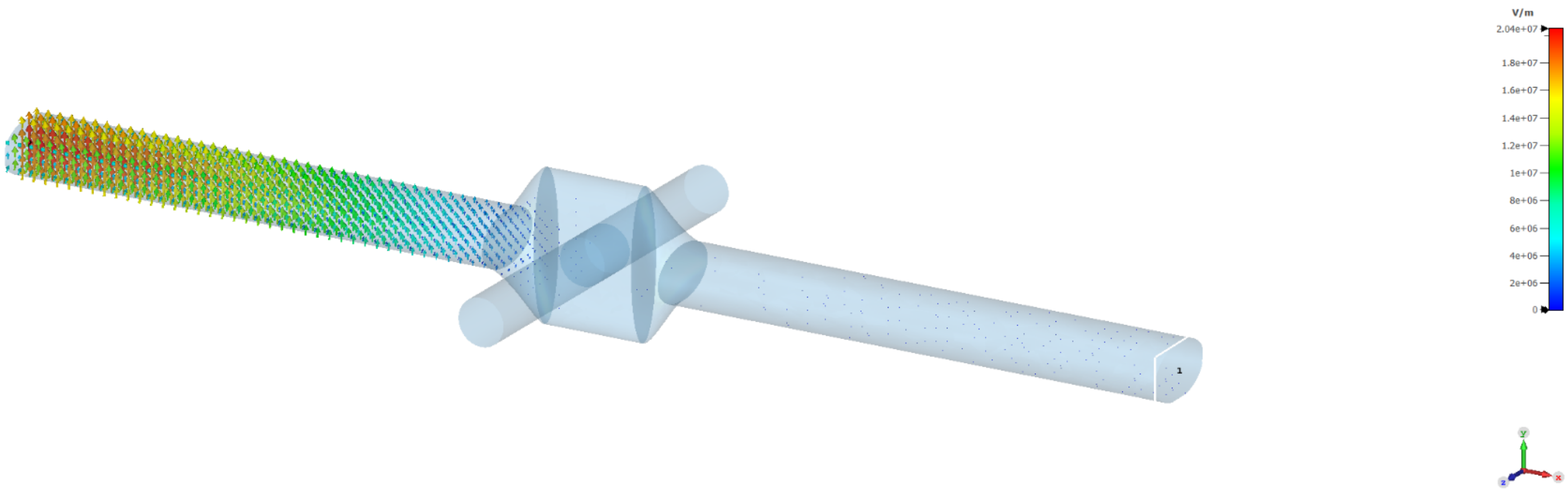


# Simulation Results

Mode 1




Mode 2



# Conclusion and Outlook

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- With proper parameter choice all combinations seem to be tuneable to work as single-mode-cavity
- Difficulties achieving a high Q and a high transit time factor remain
- But a high spacing between the ground resonance frequency and the first higher order mode frequency is possible

- Next steps:
  - Simulation with proper dampener at the waveguides ends
  - Sensitivity analysis for robust optimization
  - Try other inserts (short circuit plates, nose cones)
  - Investigate the boundary conditions for the “cavity”