

# Increasing $Q_{\text{ext}}$ of a TESLA 1.3 GHz Cavity Without Modifying the Coupler



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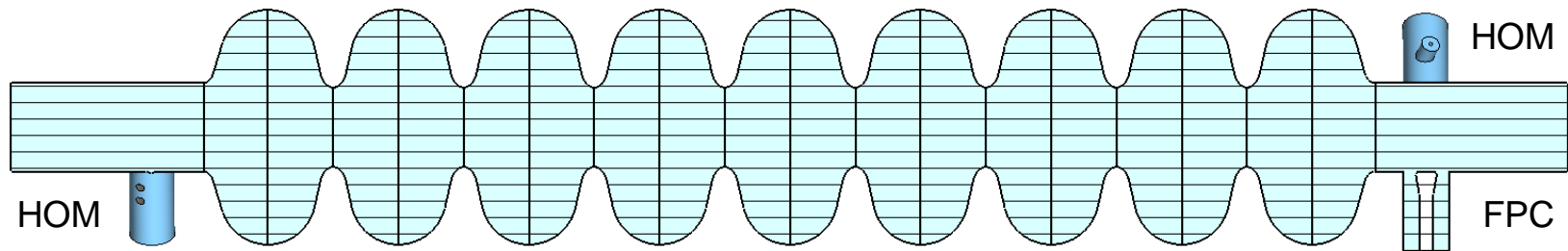


- Motivation
- Computational Modeling
  - Increasing  $Q_{\text{ext}}$  with the help of an additional scatterer
  - Determination of the corresponding scattering matrix
- Numerical Results
  - Excitation and fields in the cavity
  - Single-particle tracking
  - Horizontal and vertical coupler kicks
- Summary / Outlook

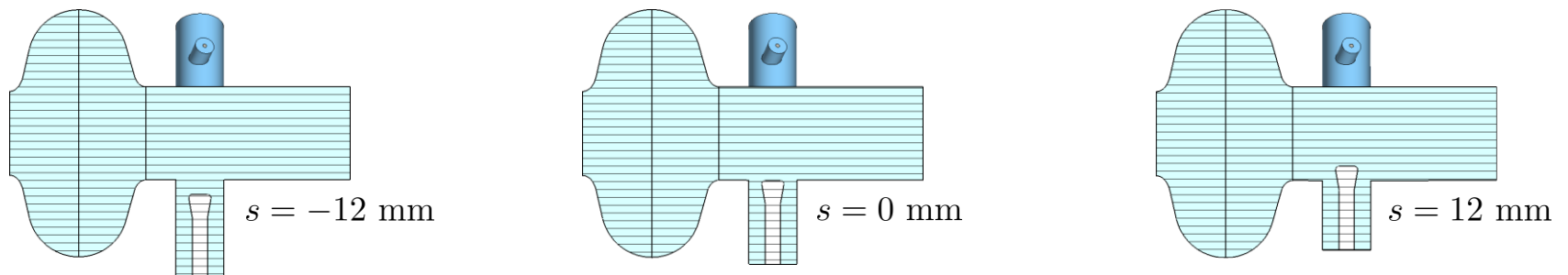
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# Motivation

- 9-Cell TESLA 1.3 GHz Cavity
  - Fundamental Setup including HOM and FPC

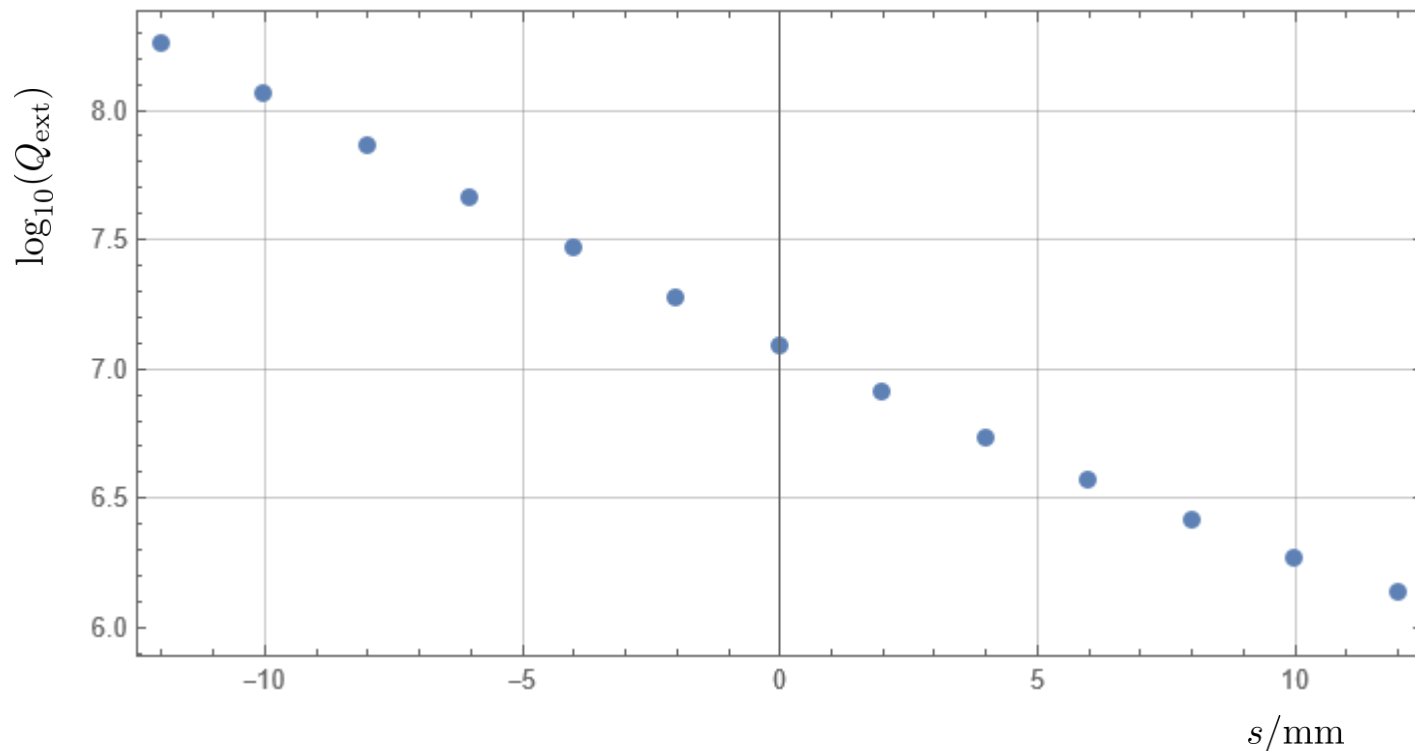


- Penetration-Depth Variation of the FPC Antenna



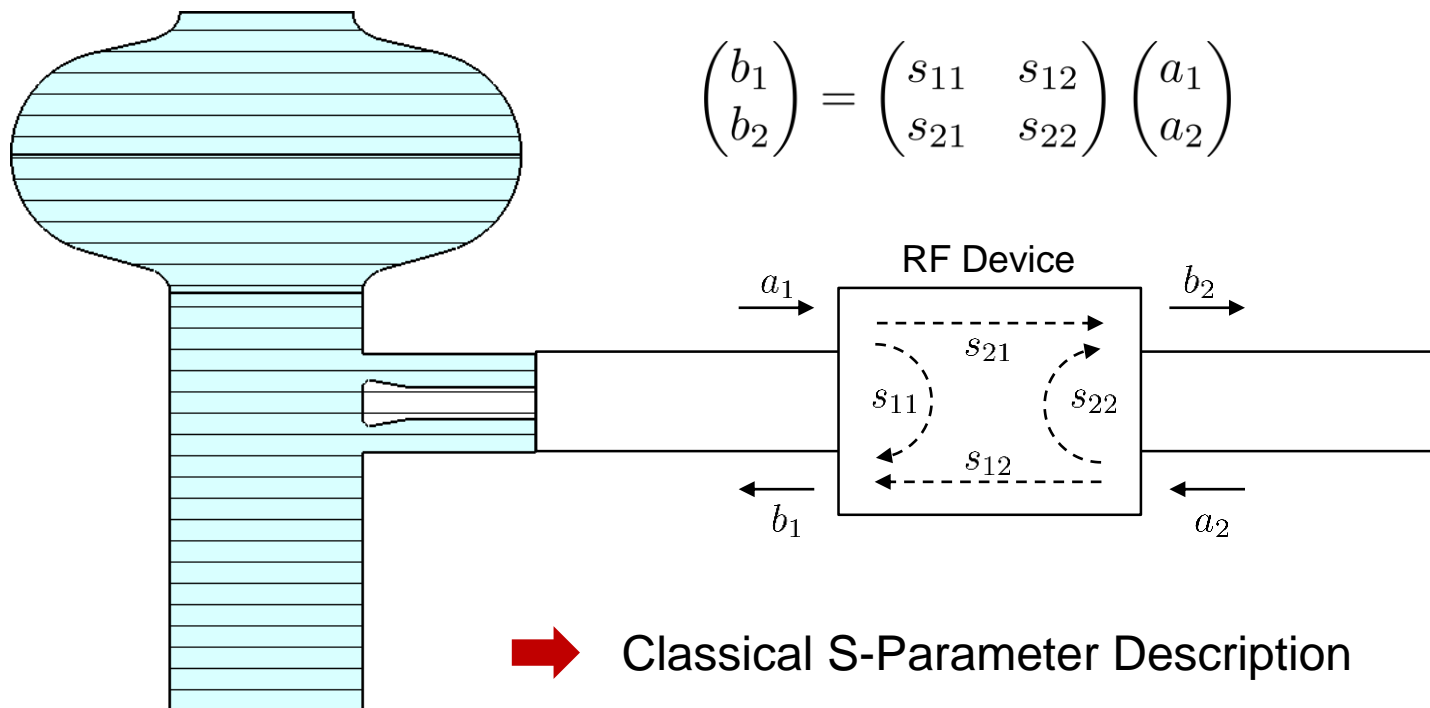
# Motivation

- 9-Cell TESLA 1.3 GHz Cavity
  - External Quality Factor (numerical solution)



- Motivation
- **Computational Modeling**
  - Increasing  $Q_{\text{ext}}$  with the help of an additional scatterer
  - Determination of the corresponding scattering matrix
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- Summary / Outlook

- 9-Cell TESLA 1.3 GHz Cavity
  - Increasing the External Quality Factor



## ▪ Properties of the Scattering Matrix

- **Symmetric** if only passive components are involved (no magnetized ferrites or plasmas)

$$S^T = S$$

$$\rightarrow s_{12} = s_{21}$$

$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

$$S^T = \begin{pmatrix} s_{11} & s_{21} \\ s_{12} & s_{22} \end{pmatrix}$$

- **Unitary** if passive and lossless

$$S^H = S^{-1}$$

$$S^H = \begin{pmatrix} s_{11}^* & s_{21}^* \\ s_{12}^* & s_{22}^* \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} s_{11}^* & s_{21}^* \\ s_{12}^* & s_{22}^* \end{pmatrix} \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$s = s_{\text{re}} + i s_{\text{im}}$$

$$s^* = s_{\text{re}} - i s_{\text{im}}$$



## ▪ Properties of the Scattering Matrix

- Unitary if passive and lossless

$$\begin{pmatrix} s_{11}^* & s_{21}^* \\ s_{12}^* & s_{22}^* \end{pmatrix} \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rightarrow s_{11}^* s_{11} + s_{21}^* s_{21} = 1$$

$$s_{11}^* s_{12} + s_{21}^* s_{22} = 0$$

$$s_{12}^* s_{11} + s_{22}^* s_{21} = 0$$

$$s_{12}^* s_{12} + s_{22}^* s_{22} = 1$$

$$\rightarrow |s_{11}|^2 + |s_{21}|^2 = 1$$

$$s_{11}^* s_{12} = -s_{21}^* s_{22}$$

$$s_{11} s_{12}^* = -s_{21} s_{22}^*$$

$$|s_{12}|^2 + |s_{22}|^2 = 1$$

$$s_{12} = s_{21}$$

$$\rightarrow |s_{11}| |s_{12}| = |s_{21}| |s_{22}|$$

$$|s_{11}| = |s_{22}|$$

$$-\arg(s_{11}) + \arg(s_{12}) = \pi - \arg(s_{21}) + \arg(s_{22})$$

$$\arg(s_{21}) = \frac{\pi}{2} + \frac{1}{2}(\arg(s_{11}) + \arg(s_{22}))$$

## ▪ Properties of the Scattering Matrix

- Symmetric if only passive components are involved

$$S^T = S$$

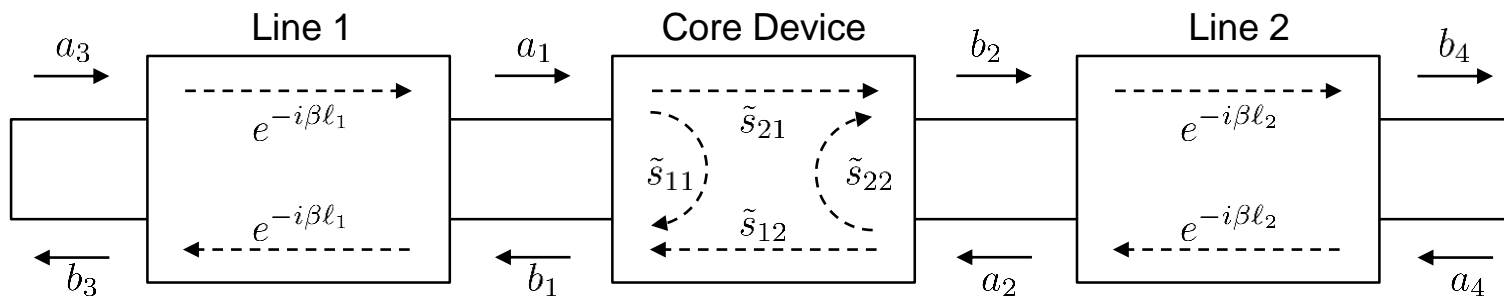
- Unitary if passive and lossless

$$S^H = S^{-1}$$

$$\rightarrow S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} \sqrt{1 - |s_{21}|^2} e^{i\varphi_1} & |s_{21}| e^{i\frac{\pi + \varphi_1 + \varphi_2}{2}} \\ |s_{21}| e^{i\frac{\pi + \varphi_1 + \varphi_2}{2}} & \sqrt{1 - |s_{21}|^2} e^{i\varphi_2} \end{pmatrix}$$

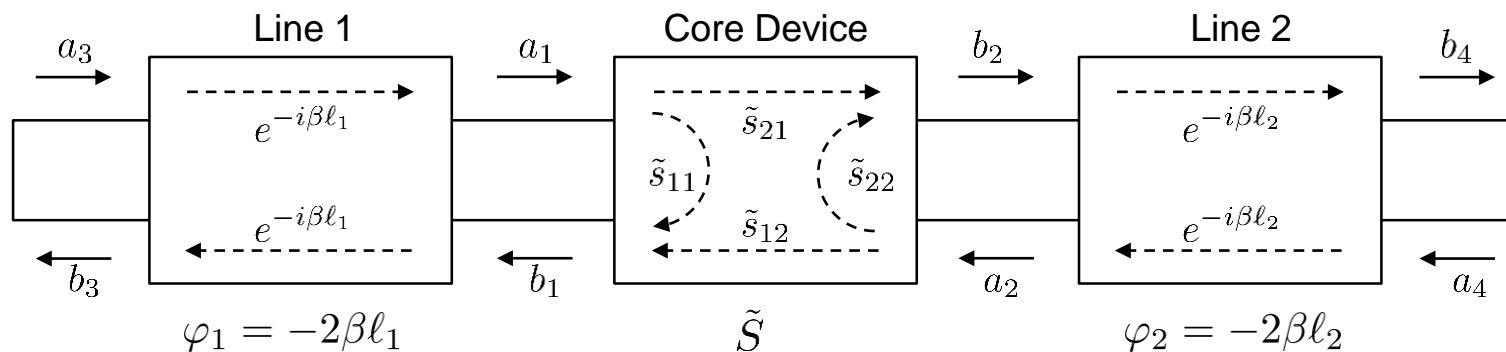
general form of the symmetric and unitary scattering matrix (3 DoF)

- Transformation of the Scattering Matrix
  - Concatenation using ideal transmission lines



$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} \sqrt{1 - |s_{21}|^2} e^{i\varphi_1} & |s_{21}| e^{i\frac{\pi + \varphi_1 + \varphi_2}{2}} \\ |s_{21}| e^{i\frac{\pi + \varphi_1 + \varphi_2}{2}} & \sqrt{1 - |s_{21}|^2} e^{i\varphi_2} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \\
 = \begin{pmatrix} \tilde{s}_{11} e^{-i\beta l_1} e^{-i\beta l_1} & \tilde{s}_{12} e^{-i\beta l_1} e^{-i\beta l_2} \\ \tilde{s}_{21} e^{-i\beta l_2} e^{-i\beta l_1} & \tilde{s}_{22} e^{-i\beta l_2} e^{-i\beta l_2} \end{pmatrix} \begin{pmatrix} a_3 \\ a_4 \end{pmatrix}$$

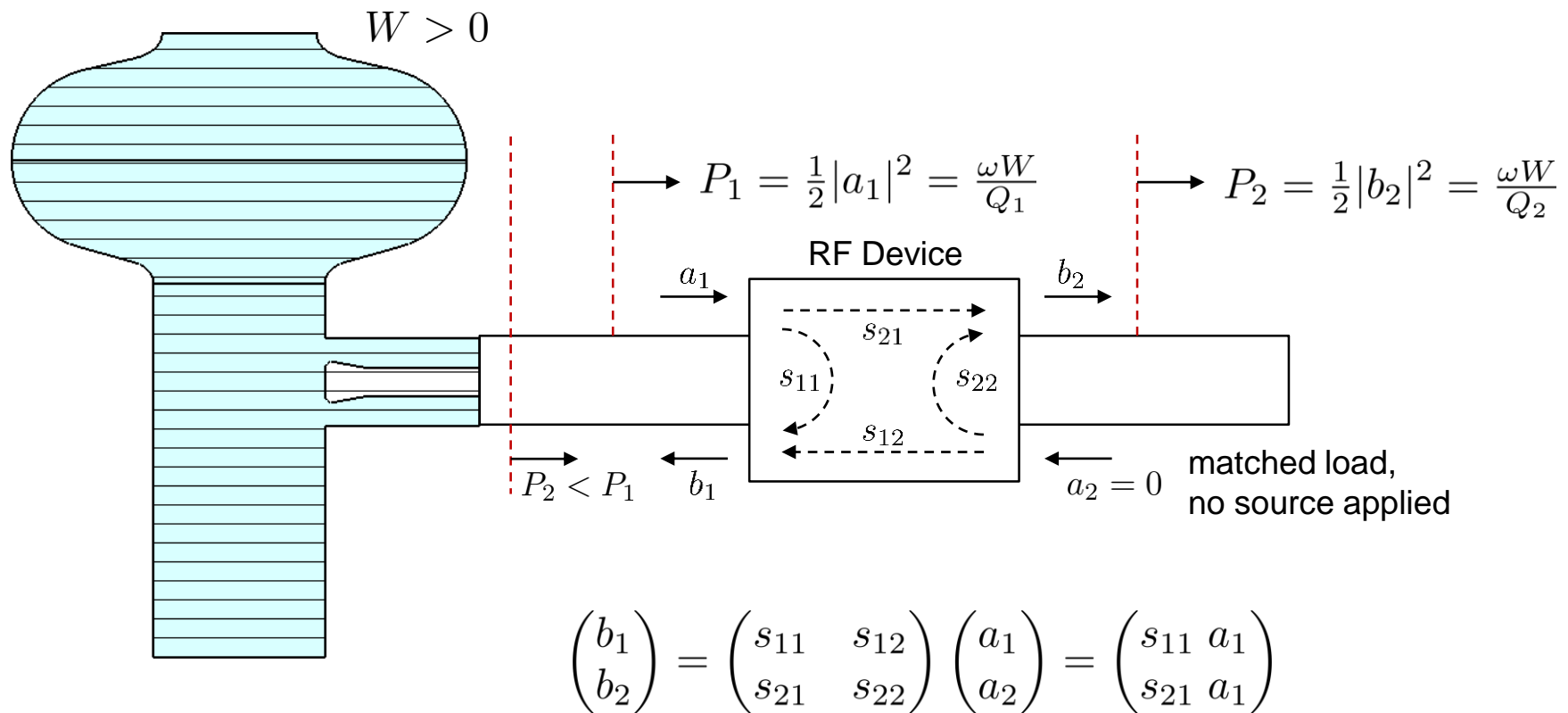
- Transformation of the Scattering Matrix
  - Concatenate using ideal lines



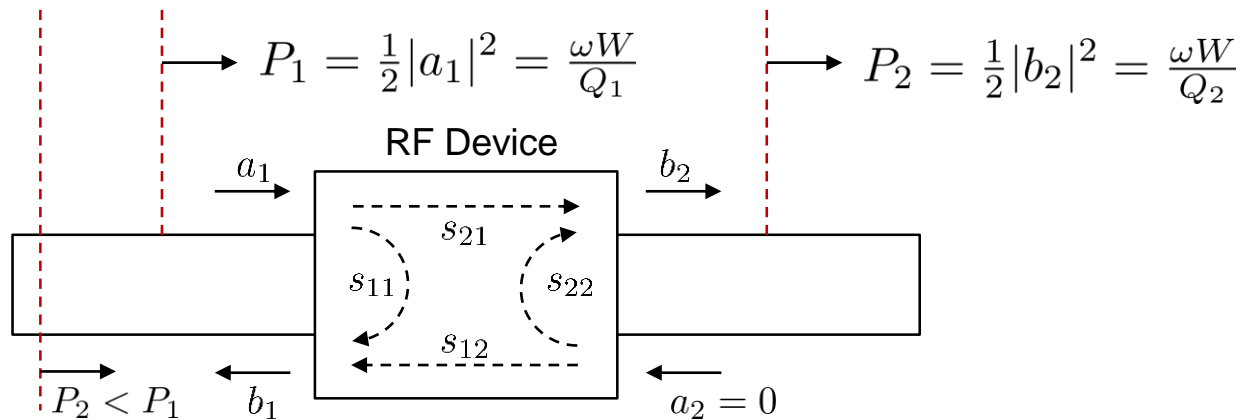
$$\tilde{S} = \begin{pmatrix} \sqrt{1 - |s_{21}|^2} & i |s_{21}| \\ i |s_{21}| & \sqrt{1 - |s_{21}|^2} \end{pmatrix}$$

reduced form of the symmetric and unitary scattering matrix (1 DoF)

## ▪ Determination of the Reduced Scattering Matrix



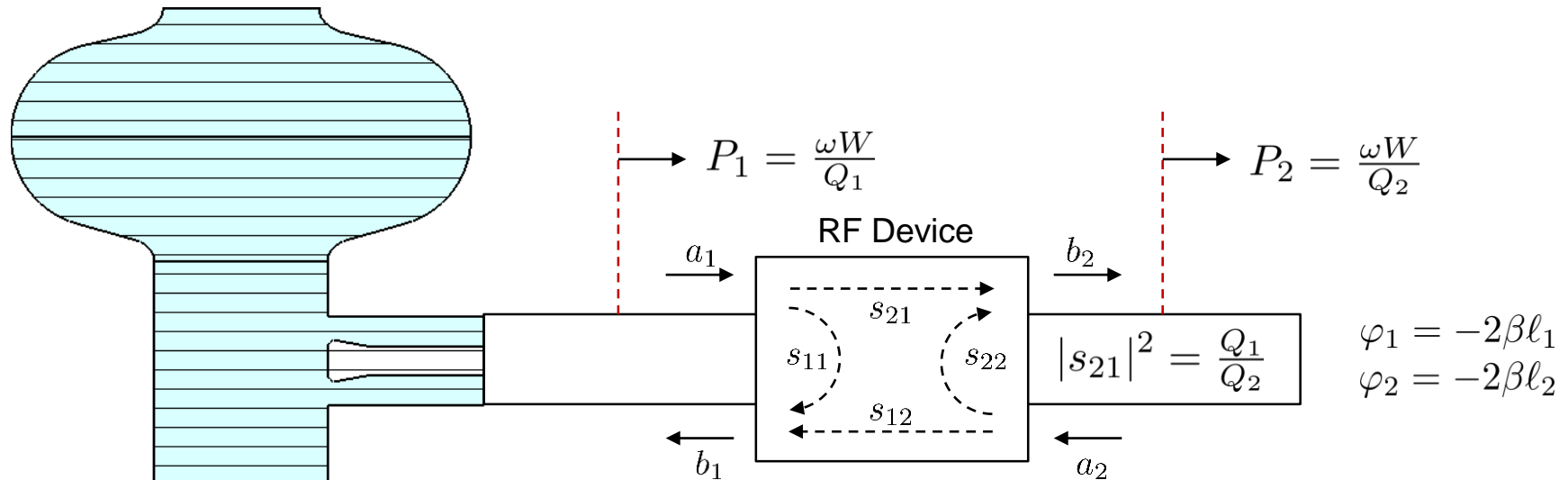
## ▪ Determination of the Reduced Scattering Matrix



$$P_2 = \frac{\omega W}{Q_2} = \frac{1}{2}|b_2|^2 = \frac{1}{2}|s_{21}a_1|^2 = \frac{1}{2}|s_{21}|^2|a_1|^2 = P_1|s_{21}|^2 = \frac{\omega W}{Q_1}|s_{21}|^2$$

$$\frac{\cancel{\omega W}}{Q_2} = \frac{\cancel{\omega W}}{Q_1}|s_{21}|^2 \quad \rightarrow \quad |s_{21}|^2 = \frac{Q_1}{Q_2} \quad Q_2 > Q_1$$

## ▪ Determination of the Scattering Matrix



$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \frac{Q_1}{Q_2}} e^{i\varphi_1} & i \sqrt{\frac{Q_1}{Q_2}} e^{i\frac{\varphi_1 + \varphi_2}{2}} \\ i \sqrt{\frac{Q_1}{Q_2}} e^{i\frac{\varphi_1 + \varphi_2}{2}} & \sqrt{1 - \frac{Q_1}{Q_2}} e^{i\varphi_2} \end{pmatrix}$$

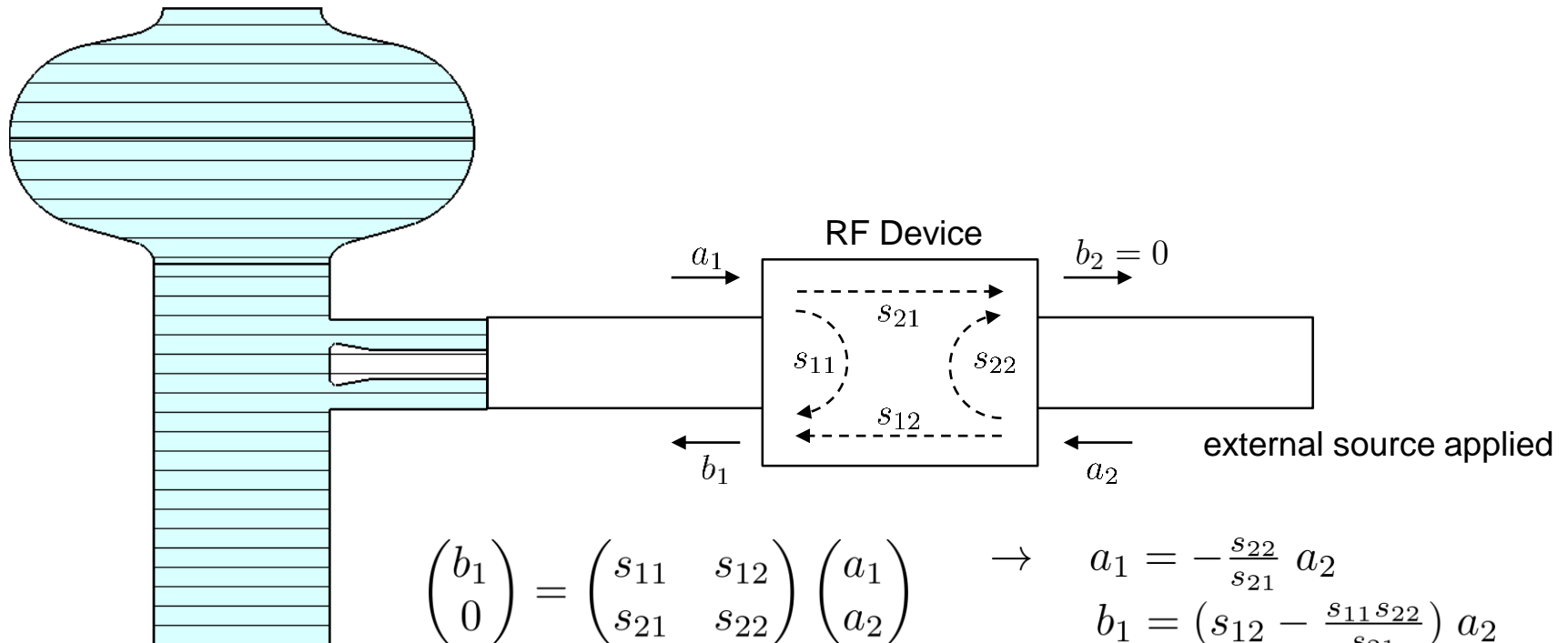
symmetric and unitary scattering matrix

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- Computational Modeling
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- **Numerical Results**
  - Excitation and fields in the cavity
  - Single-particle tracking
  - Horizontal and vertical coupler kicks
- Summary / Outlook



# Numerical Results

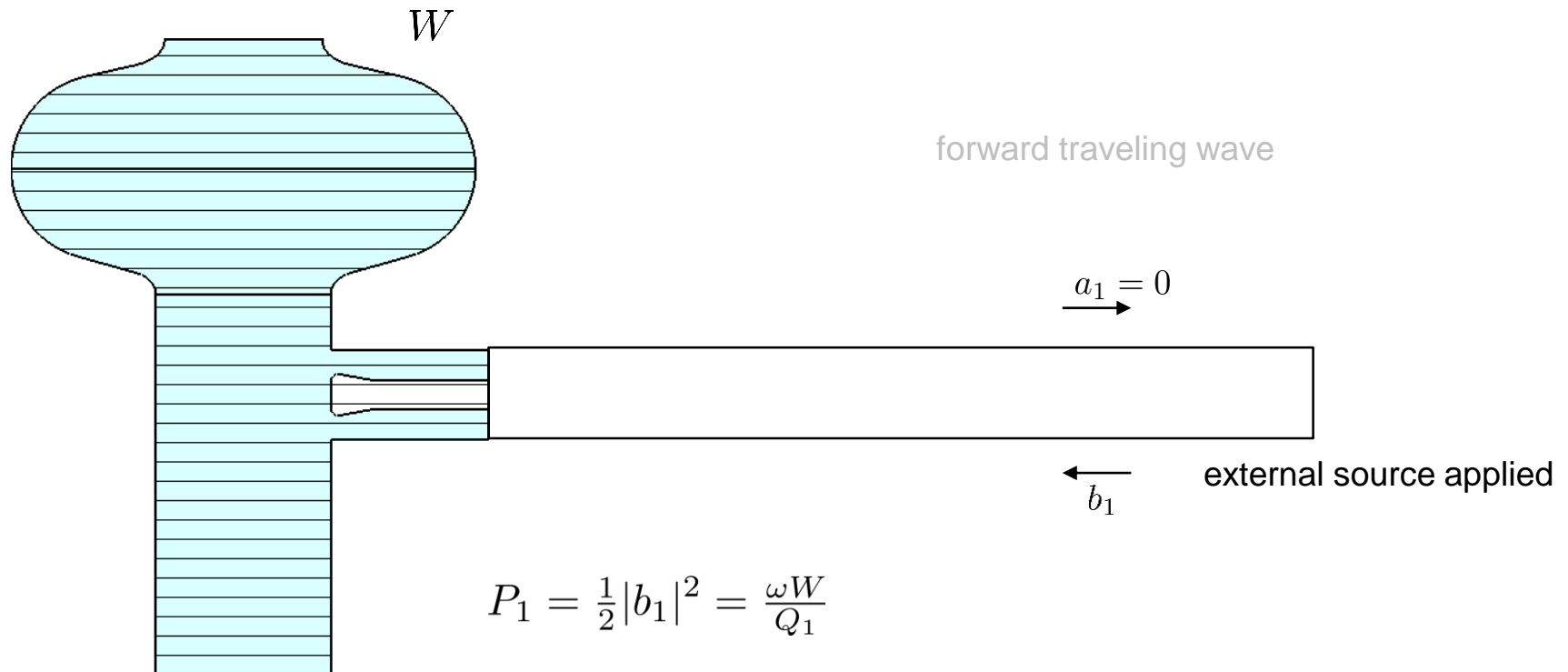
- Excitation of the Cavity through the RF Device



$$\begin{pmatrix} b_1 \\ 0 \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \rightarrow \begin{aligned} a_1 &= -\frac{s_{22}}{s_{21}} a_2 \\ b_1 &= \left( s_{12} - \frac{s_{11}s_{22}}{s_{21}} \right) a_2 \end{aligned}$$
$$\rightarrow \frac{a_1}{b_1} = s_{11}^*$$

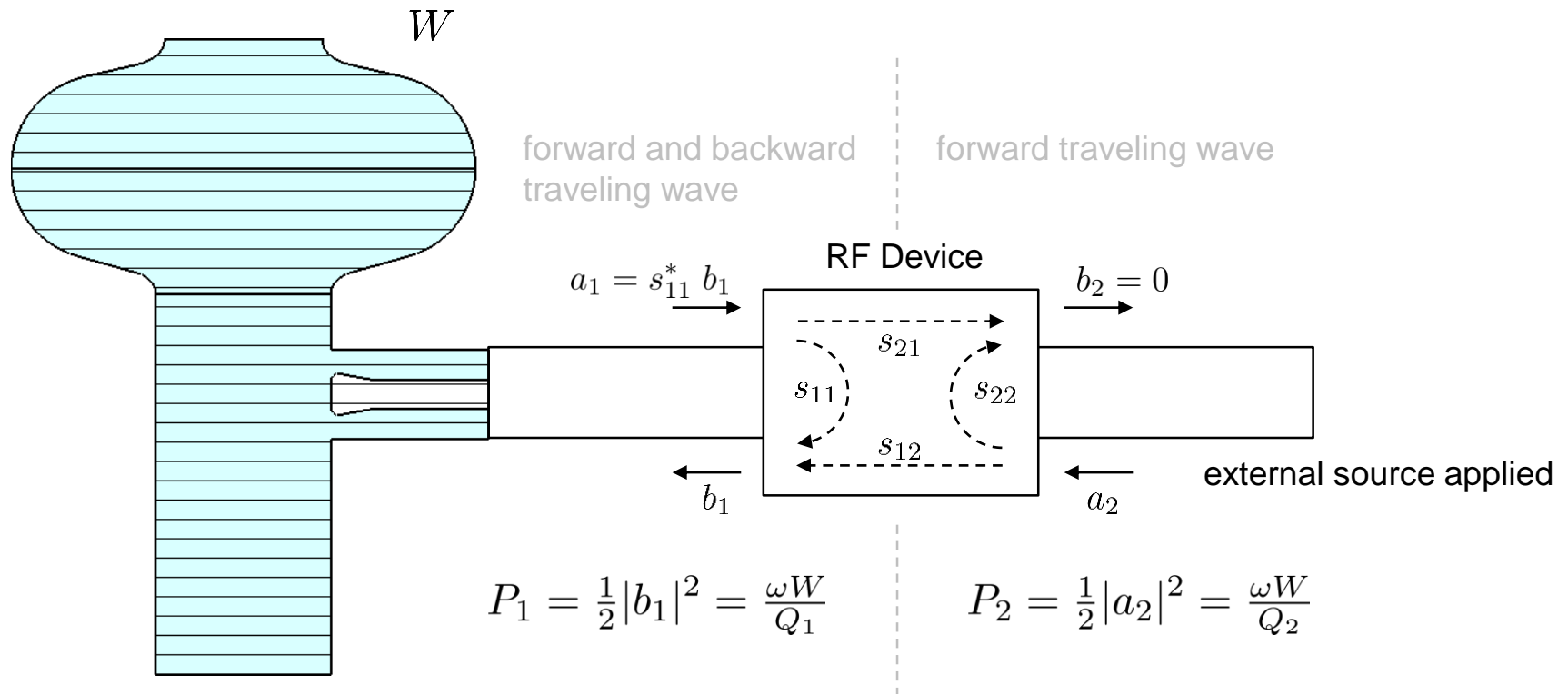
# Numerical Results

- Comparison of the Different Coupling Schemes
  - Direct insertion of the applied power into the cavity



# Numerical Results

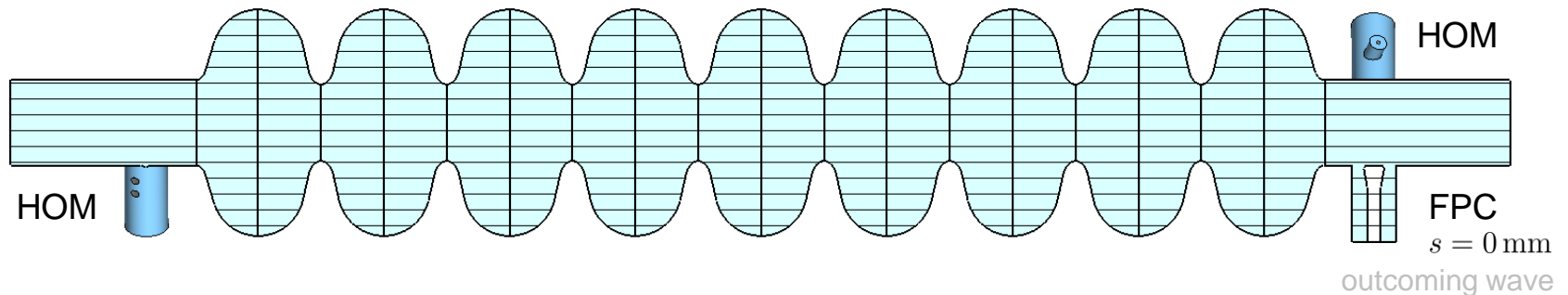
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# Numerical Results

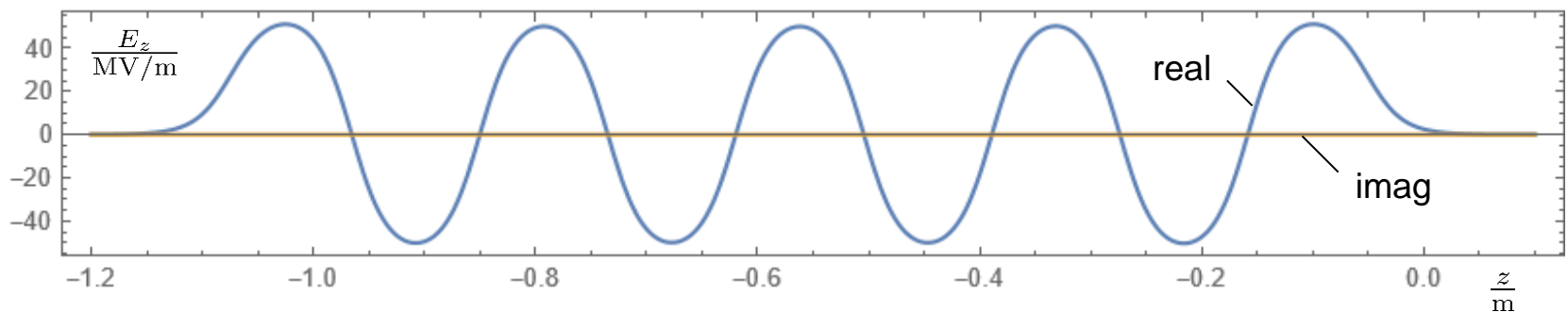
## ▪ 9-Cell TESLA 1.3 GHz Cavity

- Fundamental Setup including HOM and FPC



- Calculate Fields on Axis ( $E_x$ ,  $E_y$ ,  $E_z$ ,  $B_x$ ,  $B_y$ ,  $B_z$ )

$$E_z|_{\max} \stackrel{!}{=} 50 \frac{\text{MV}}{\text{m}}$$

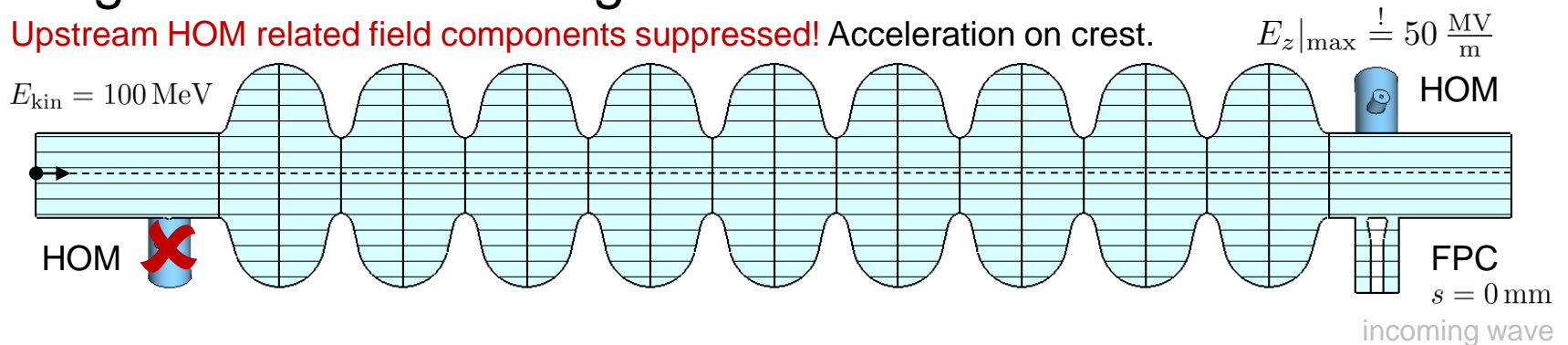


# Numerical Results

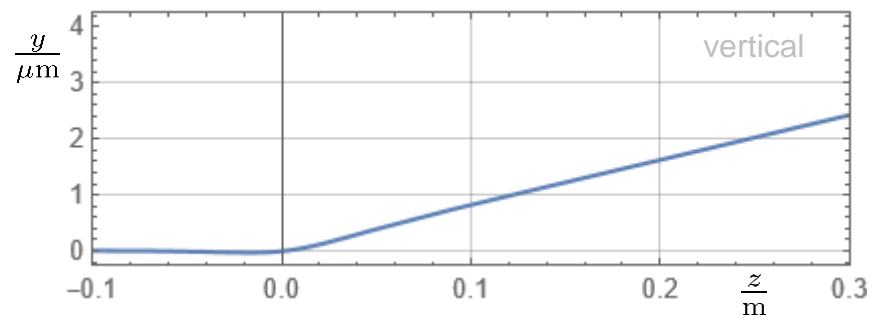
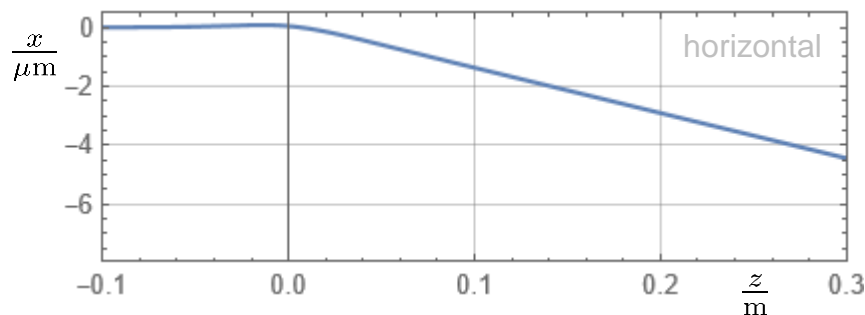
## ▪ 9-Cell TESLA 1.3 GHz Cavity

### - Single-Particle Tracking

Upstream HOM related field components suppressed! Acceleration on crest.



### - Trajectory in the Horizontal and Vertical Planes

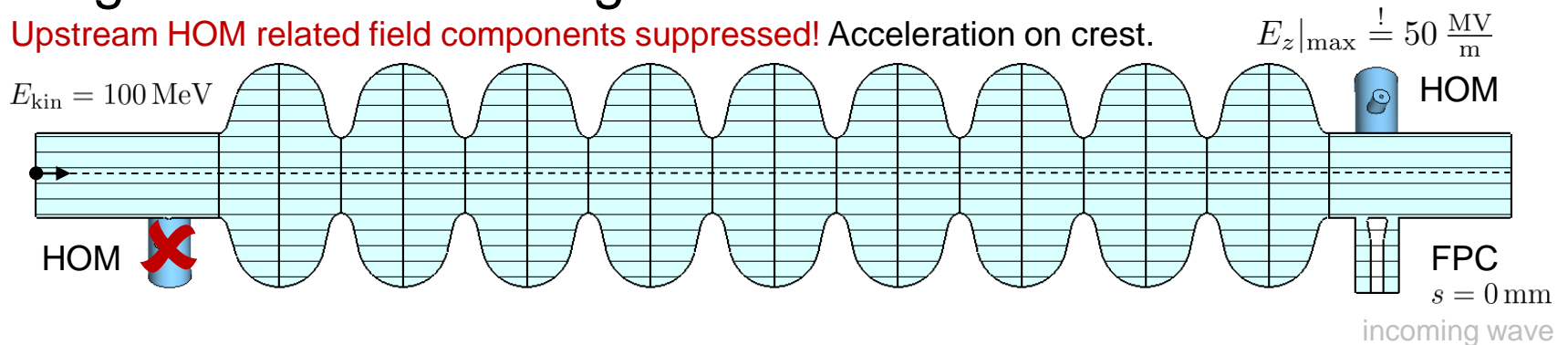


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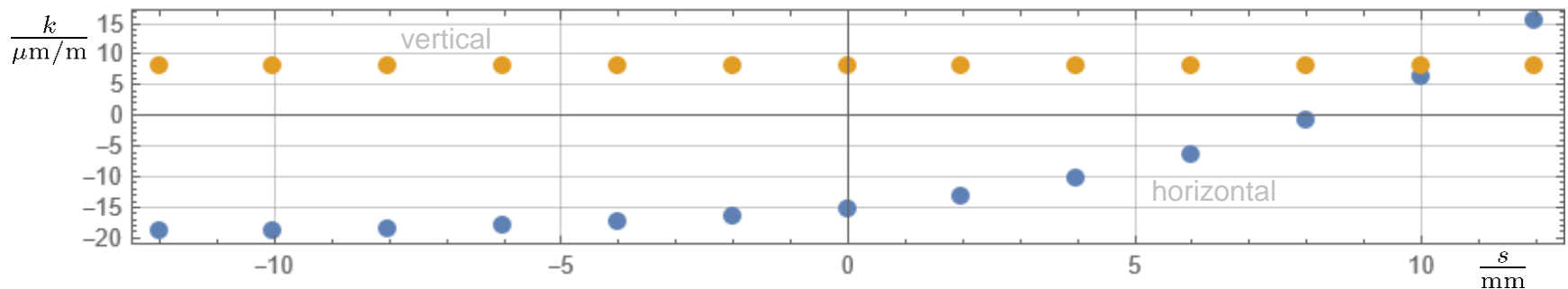
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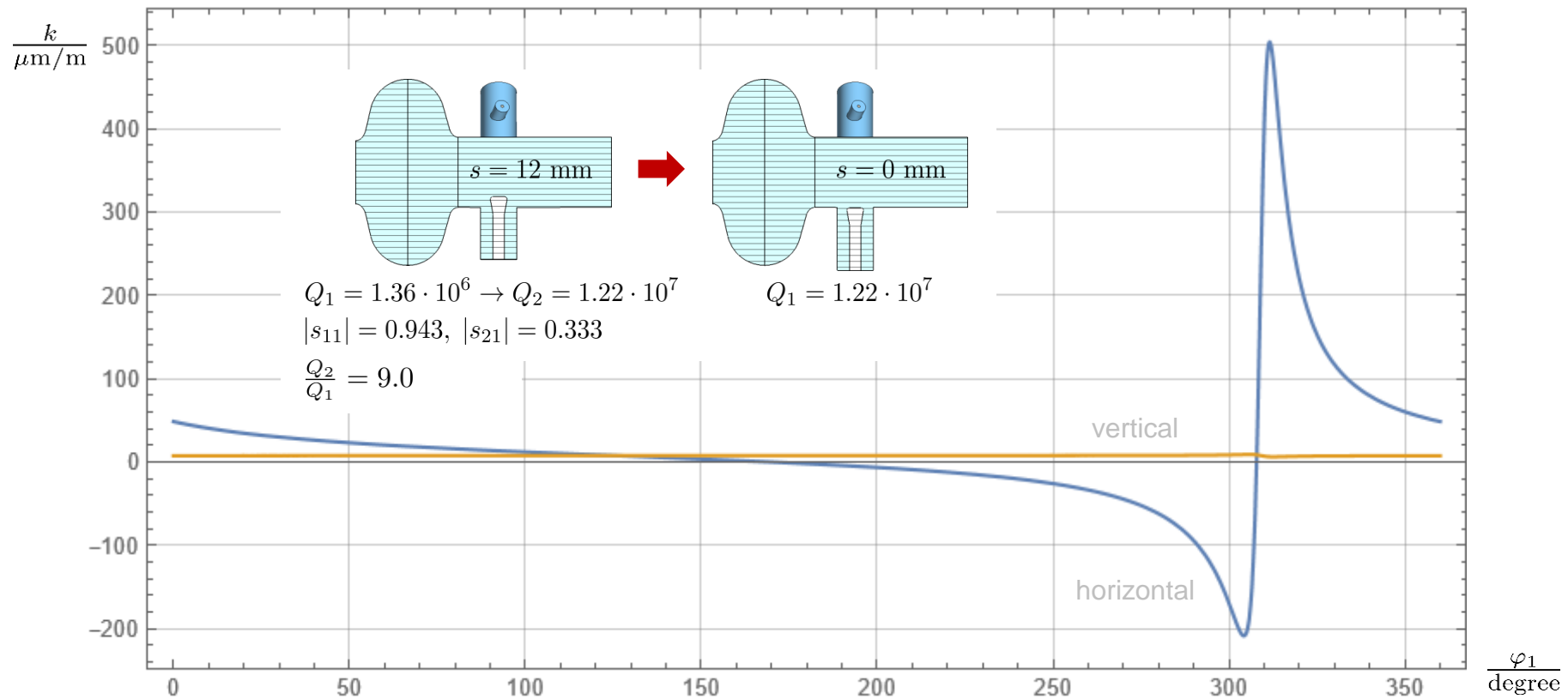


### - Trajectory in the Horizontal and Vertical Planes



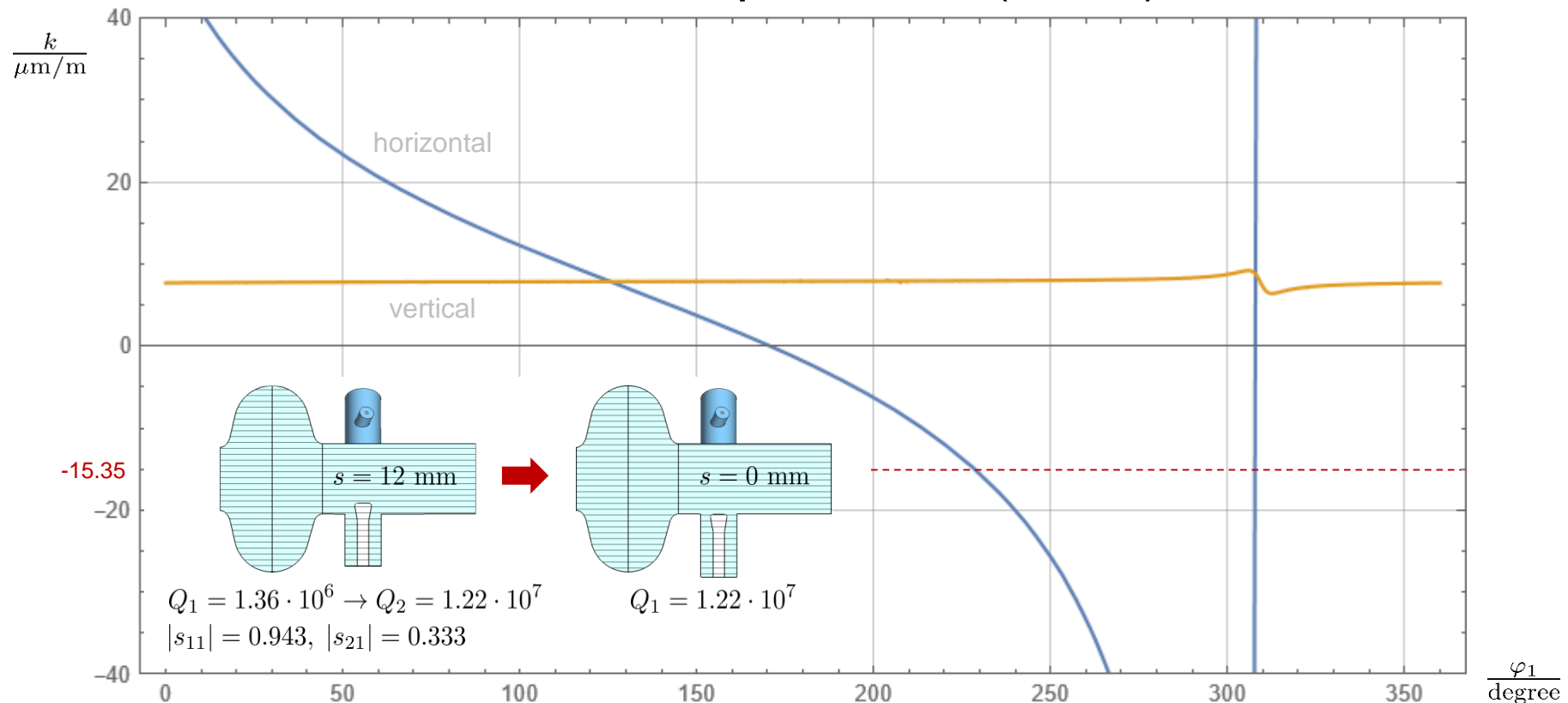
# Numerical Results

- 9-Cell TESLA 1.3 GHz Cavity
  - Horizontal and Vertical Coupler Kicks



# Numerical Results

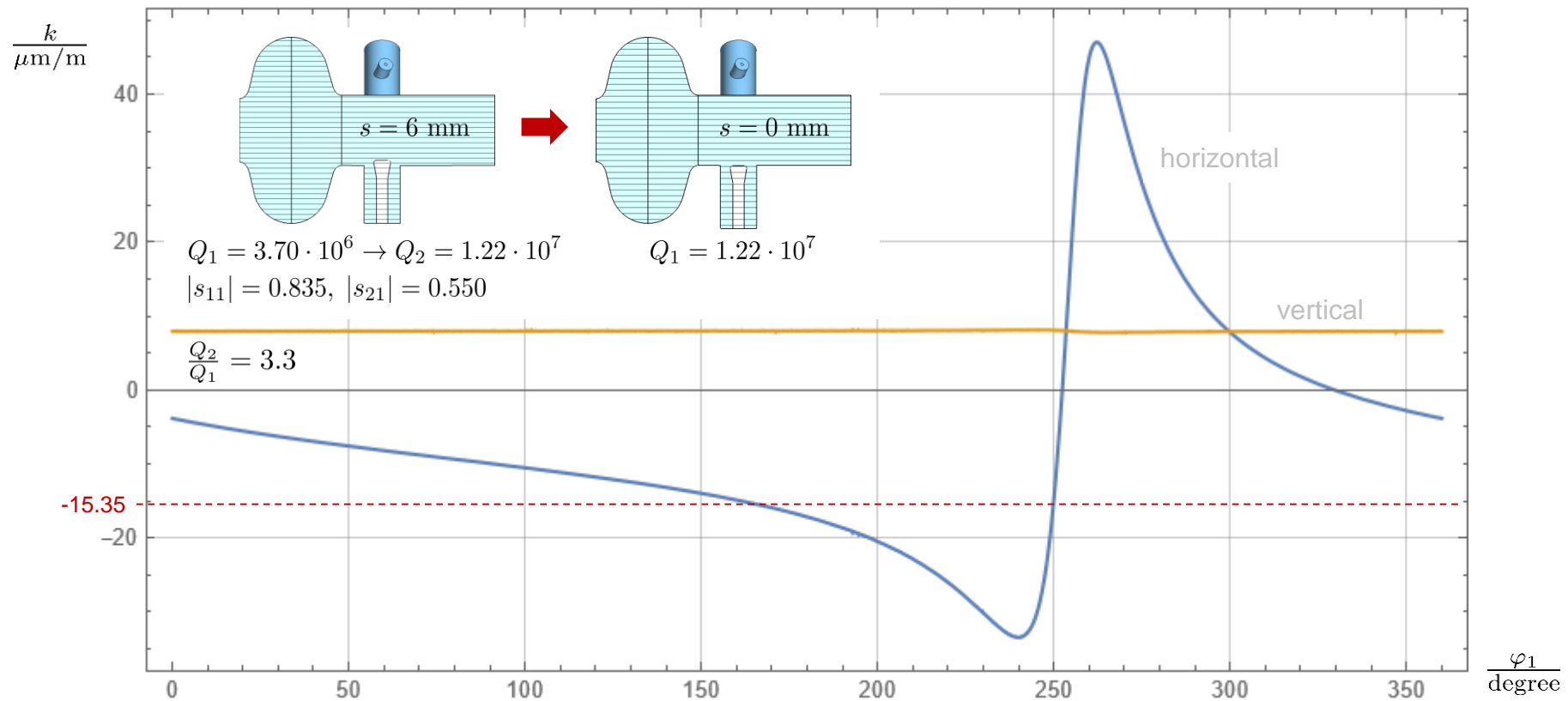
- 9-Cell TESLA 1.3 GHz Cavity
  - Horizontal and Vertical Coupler Kicks (zoom)





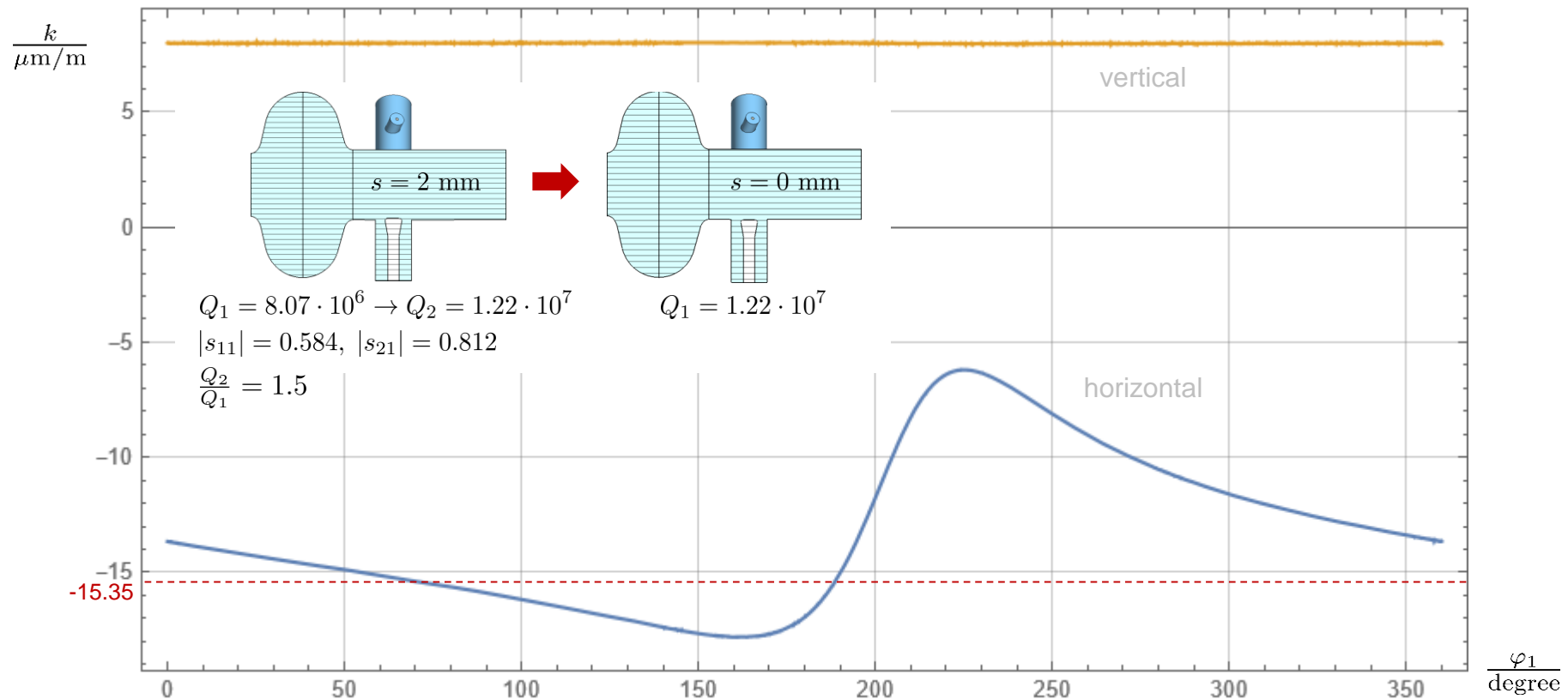
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- 9-Cell TESLA 1.3 GHz Cavity
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## ▪ Summary

- Adding a scatterer in the feeding line of the cavity
- Determination of the corresponding scattering matrix
- Feeding the cavity and calculation of the EM fields
- Single-particle dynamics
- Kick-factor calculations

➔ Even with highly reflecting scatterers moderate kick factors achievable

## ▪ Outlook

- Modeling a scatterer with the desired properties

