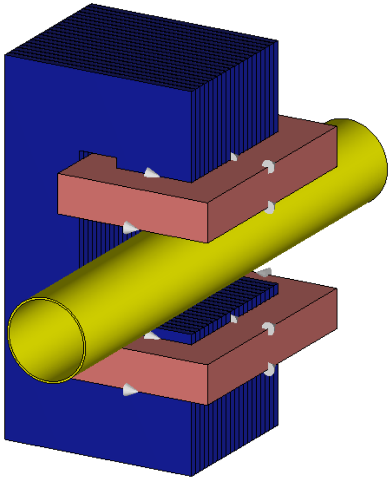
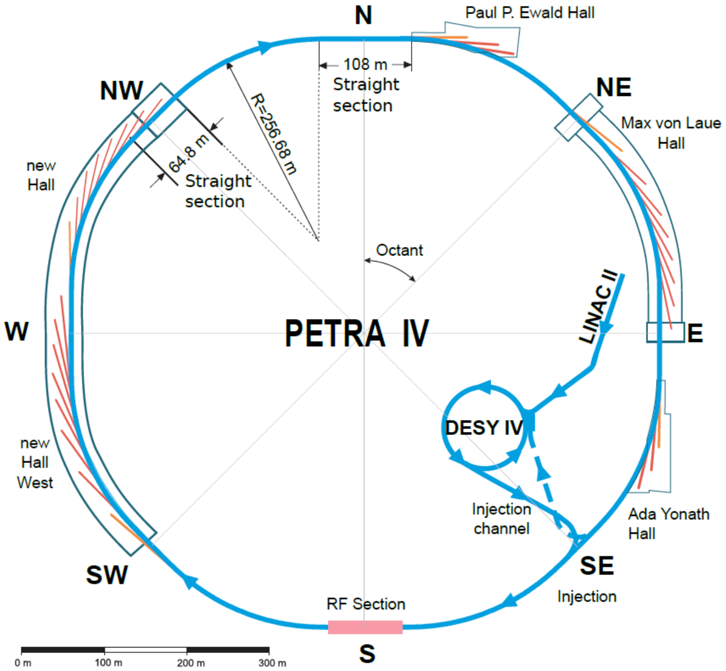
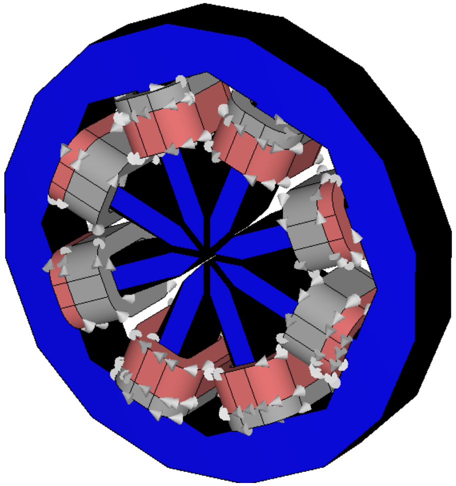
Finite-Element Simulation of Eddy-Current Effects in Orbit Corrector Magnets

Jan-Magnus Christmann





PETRA IV Conceptual Design Report



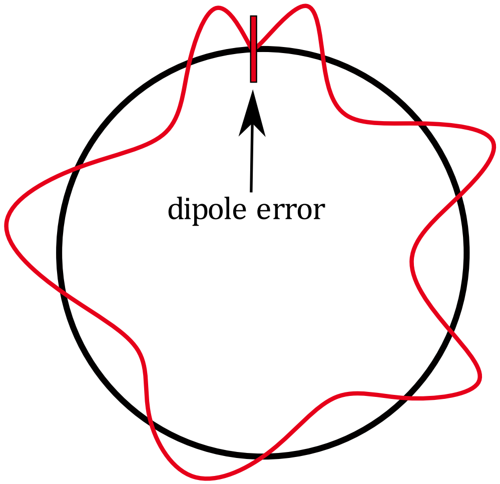
Contents

Introduction

- Circular accelerators need dipole magnets to correct orbit distortions
- **PETRA IV**: ultra-low emittance synchrotron radiation source

  ➔ **AC correctors** with **frequencies in kHz-range** necessary
- **Strong eddy currents** ➔ power losses, time delay, and field distortion
- **Simulation challenging** due to small skin depths and laminated yoke

  ➔ Need for technique to simplify simulations



Based on K. Wille, Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen

Contents

Homogenization Technique

- Magnetoquasistatic PDE: $\nabla \times (\nu \nabla \times \underline{\vec{A}}) + j\omega\sigma\underline{\vec{A}} = \underline{\vec{J}}_s$
- Adapt reluctivity ν and conductivity σ in the laminated yoke

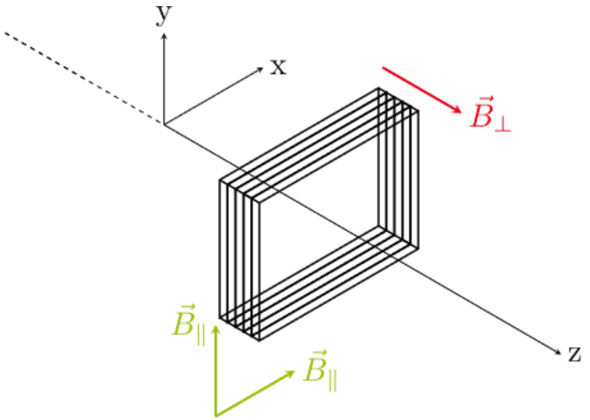
$$\nu \rightarrow \underline{\bar{\bar{\nu}}} = \frac{1}{8}\sigma_{\mathrm{Fe}} d\delta\omega(1+j)\frac{\sinh((1+j)\delta^{-1}d)}{\sinh^2\big((1+j)\delta^{-1}\,d/2\big)}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix} + \nu_{\mathrm{Fe}}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

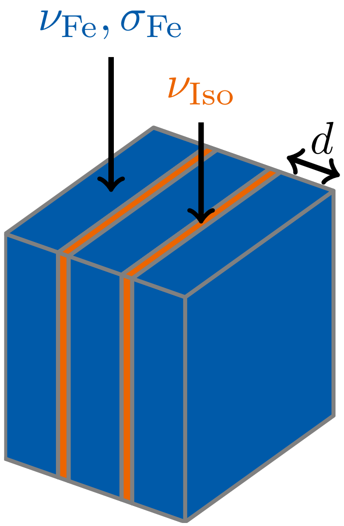
$$\sigma \rightarrow \bar{\bar{\sigma}} = \gamma\sigma_{\mathrm{Fe}}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

P. Dular et al., 2003
L. Krählenbühl et al., 2004

Skin depth $\delta = \sqrt{2/\omega\sigma_{\mathrm{Fe}}\mu_{\mathrm{Fe}}}$

Stacking factor $\gamma = \frac{V_{\mathrm{Fe}}}{V_{\mathrm{Yoke}}}$



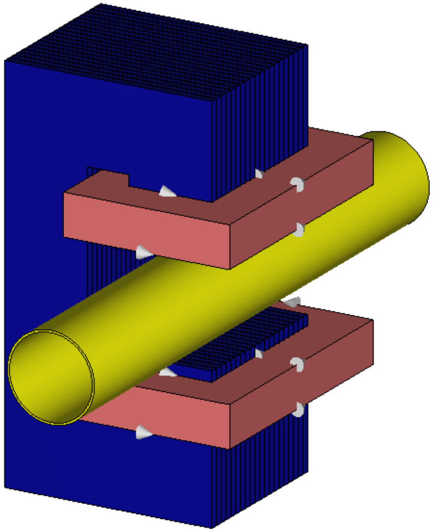


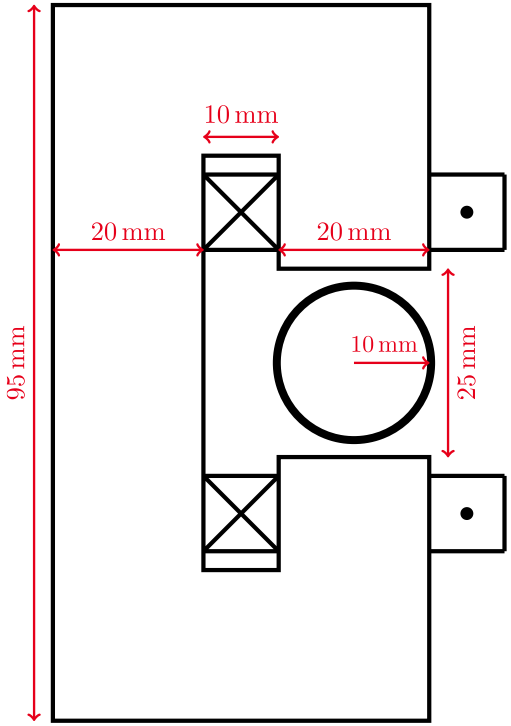
Contents

- Introduction
- Homogenization Technique
- Toy Model
- Realistic Model
- Conclusion/Outlook

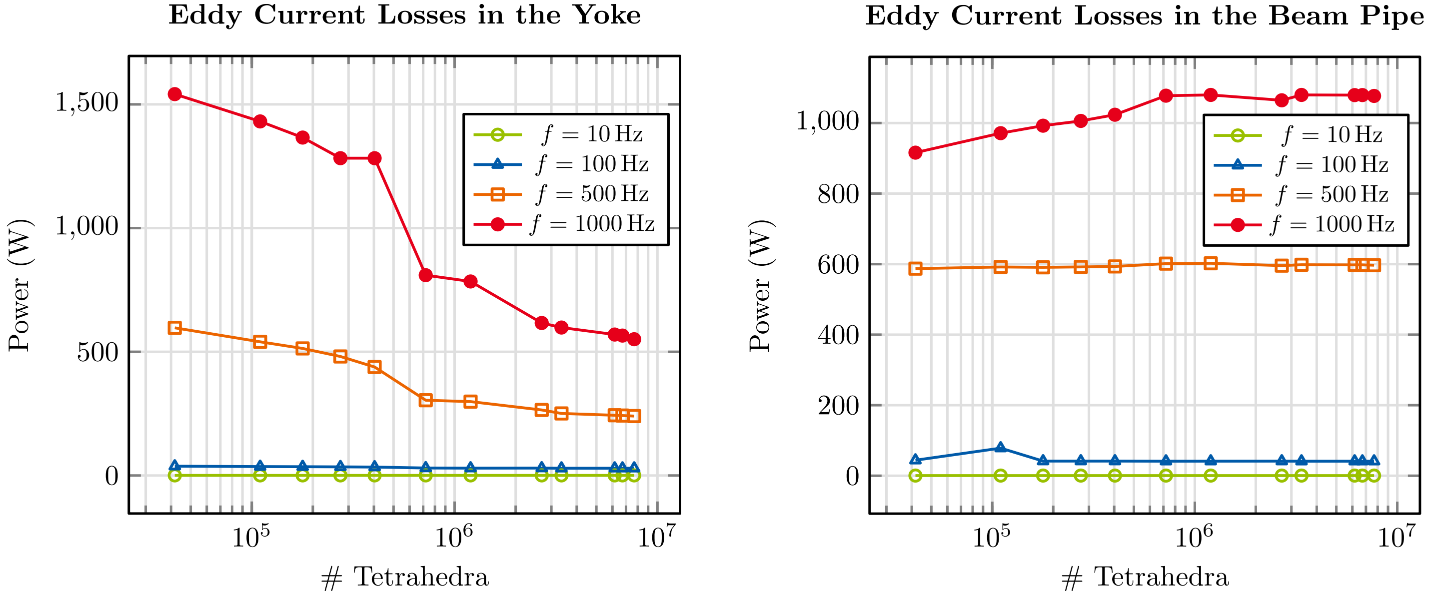
Model Description

- **Iron yoke:** length = 40 mm, lamination thickness = 1.83 mm
- **Copper beam pipe:** thickness = 0.5 mm, length = 140 mm
- **Coils**: current **=** 10 A (peak), # turns = 250
- **Frequency domain simulation** via **CST Studio Suite®**





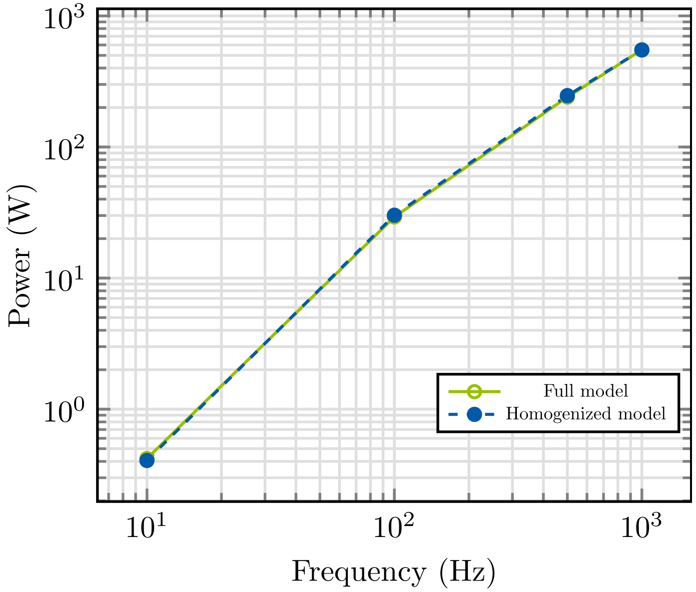
Simulation of the Full Model

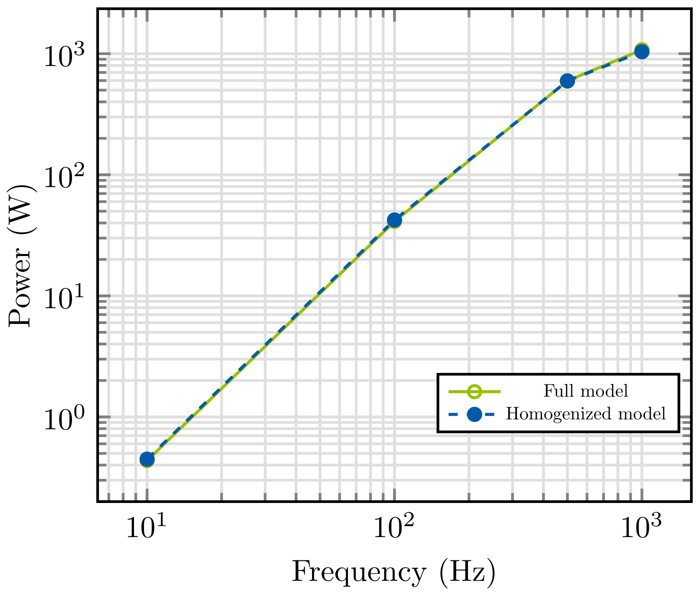


# Tetrahedra	$4.2 \cdot 10^4$	$4.0 \cdot 10^5$	$1.2 \cdot 10^6$	$3.4 \cdot 10^6$	$7.7 \cdot 10^6$
Simulation time	2 min	20 min	1 h	7.5 h	21.5 h

- Strong mesh dependence of power losses at higher frequencies
  - ➔ Obtaining reliable results is difficult
  - ➔ Need for simplified model

Homogenized vs. Full Model

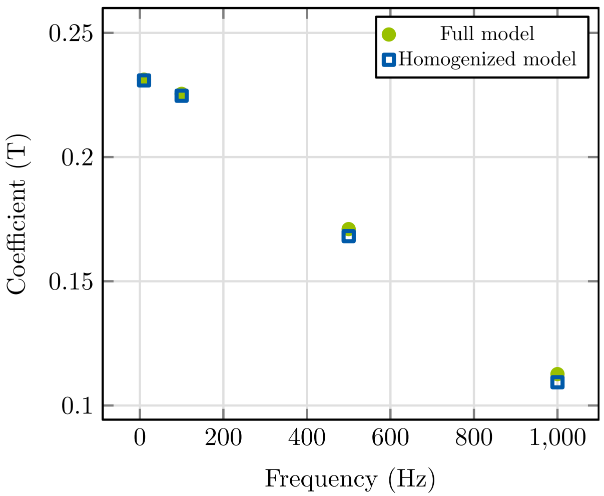

Eddy Current Losses in the Yoke


Eddy Current Losses in the Beam Pipe

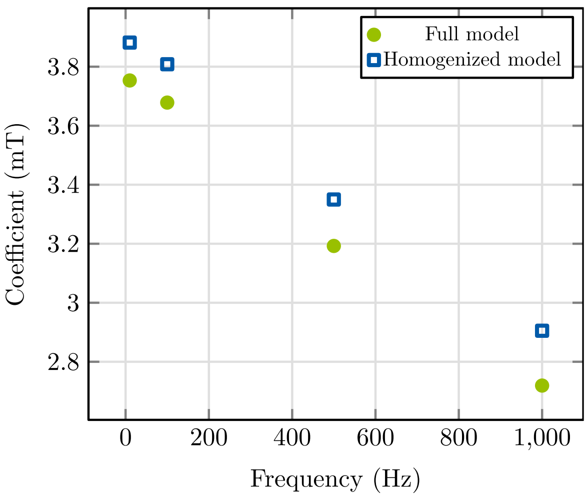
- Good approximation of losses in yoke & beam pipe (max. relative error 4 %)
- Simulation time reduced from several hours to 4 min

Homogenized vs. Full Model

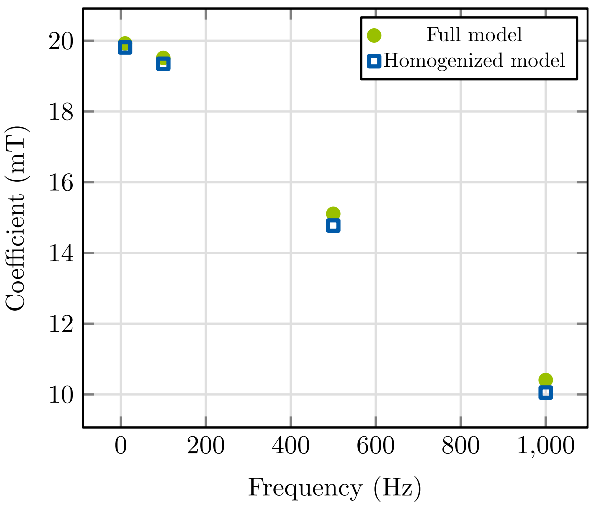
Normal Dipole Coefficients



Normal Quadrupole Coefficients



Normal Sextupole Coefficients



Multipole coefficient	**Average rel. error**
Dipole	1 %
Quadrupole	5 %
Sextupole	2 %

- Homogenization technique yields accurate multipole coefficients

➔ Aperture field accurately represented

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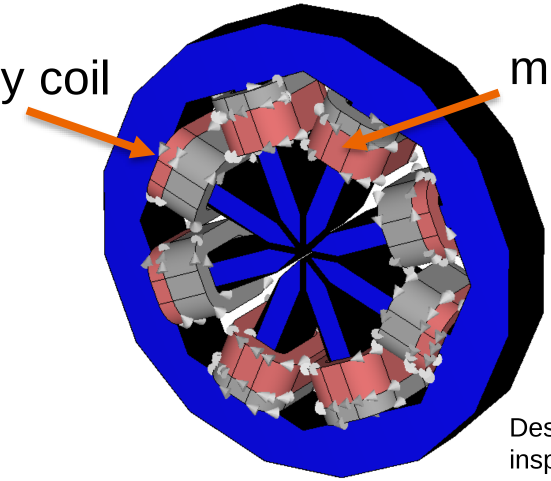
Model Description

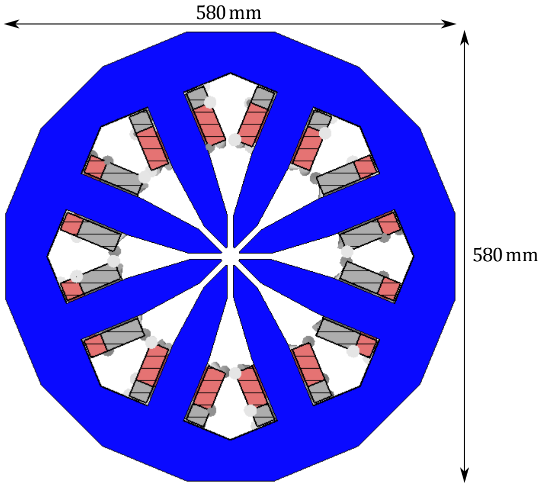
- **Dipole** corrector with **octupole-like design**
- **Coils**:
  - 4 main coils: current = 27.4 A (peak), # turns = 53
  - 4 auxiliary coils: current = 27.4 A (peak), # turns = 22
- **Iron yoke:**
  - Diameter = 580 mm, length = 90 mm
  - Lamination thickness = 0.5 mm
- At first **no beam pipe**



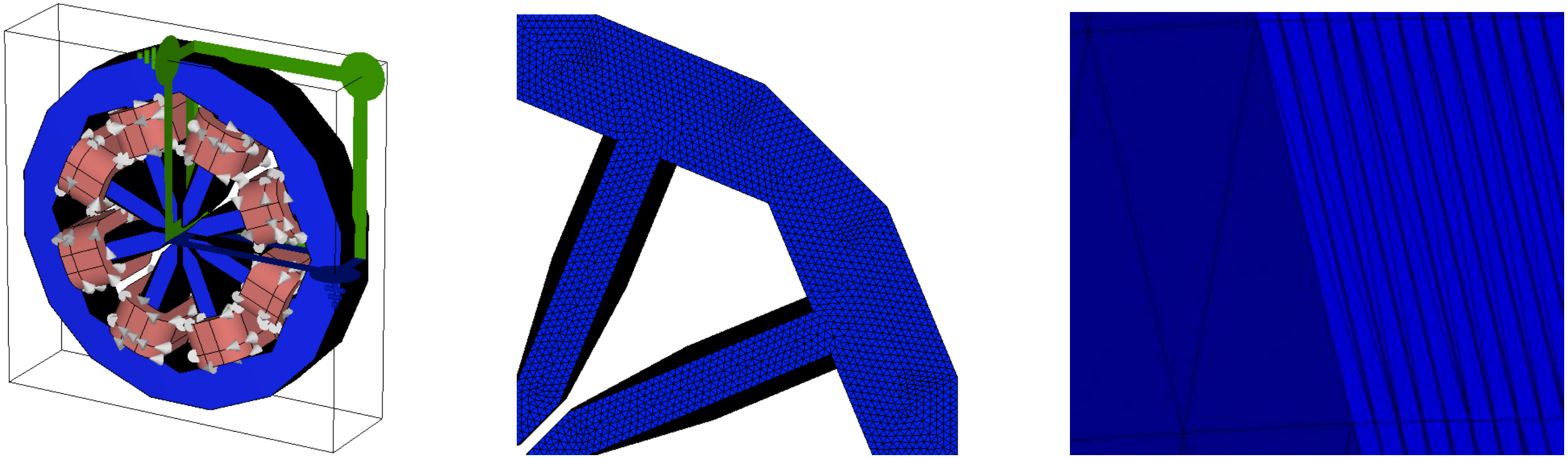
auxiliary coil

main coil

Design by A. Aloev (DESY), inspired by APS

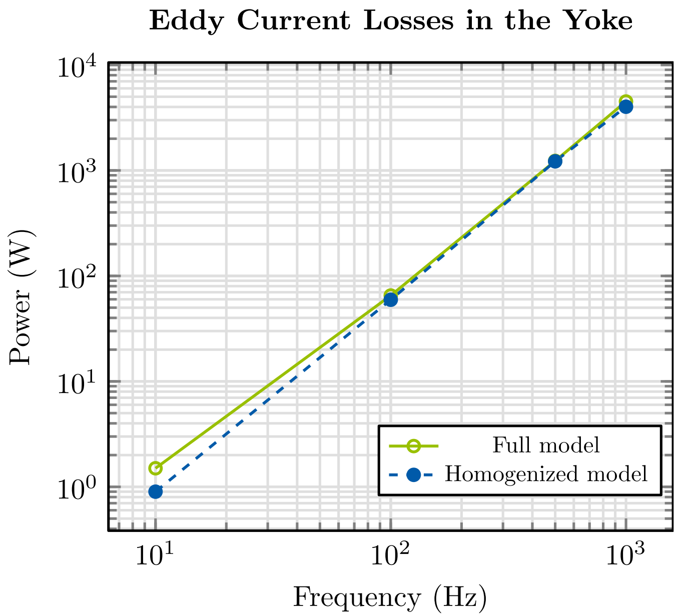


Simulation of the Full Model



- **Frequency domain simulation** via **CST Studio Suite®**
- Three symmetry planes, test frequencies $f = 10\ \mathrm{Hz}, 100\ \mathrm{Hz}, 500\ \mathrm{Hz}, 1000\ \mathrm{Hz}$
- Long simulation times even for relatively coarse meshes
- Finest mesh: # tetrahedra = $2.3 \cdot 10^6$ ➔ simulation time = 26 h
- Skin depth cannot be resolved ➔ power loss still mesh-dependent

Homogenized vs. Full Model



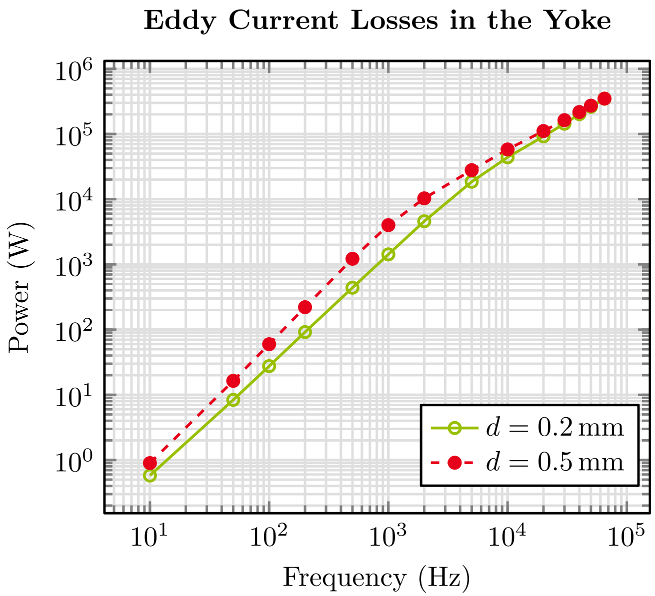
Multipole coefficient	Average rel. deviation
Dipole	1 %
14-pole	1 %
18-pole	3 %

- Similar power losses
- Good agreement in multipole coefficients
- Simulation time reduces from 26 h to 5 min

➔ Homogenized model can be used for further studies

Keep in mind:
Power losses in full model are still mesh-dependent !

Power Loss for Different Lamination Thicknesses


Eddy Current Losses in the Yoke

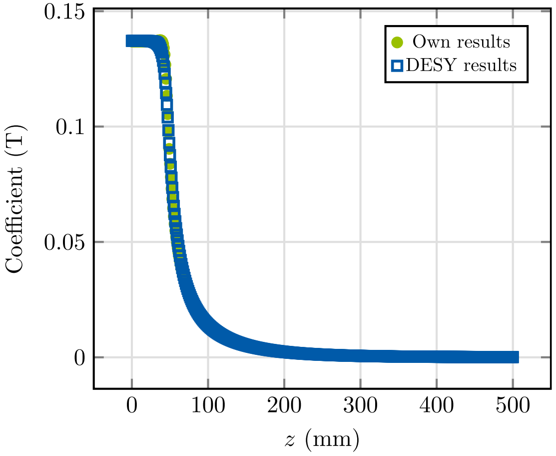
f(Hz)	Eddy current losses (W)			
	$d = 0.2$ mm	$d = 0.3$ mm	$d = 0.4$ mm	$d = 0.5$ mm
10	$5.8 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$	$7.6 \cdot 10^{-1}$	$9.0 \cdot 10^{-1}$
100	$2.8 \cdot 10^{1}$	$3.4 \cdot 10^{1}$	$4.6 \cdot 10^{1}$	$6.0 \cdot 10^{1}$
500	$4.4 \cdot 10^{2}$	$6.2 \cdot 10^{2}$	$9.0 \cdot 10^{2}$	$1.2 \cdot 10^{3}$
1000	$1.4 \cdot 10^{3}$	$2.1 \cdot 10^{3}$	$3.1 \cdot 10^{3}$	$4.0 \cdot 10^{3}$
10000	$4.4 \cdot 10^{4}$	$4.9 \cdot 10^{4}$	$5.5 \cdot 10^{4}$	$5.8 \cdot 10^{4}$
30000	$1.4 \cdot 10^{5}$	$1.6 \cdot 10^{5}$	$1.6 \cdot 10^{5}$	$1.6 \cdot 10^{5}$
65000	$3.5 \cdot 10^{5}$	$3.6 \cdot 10^{5}$	$3.6 \cdot 10^{5}$	$3.5 \cdot 10^{5}$

- Use homogenization to investigate losses up to $65\ \mathrm{kHz}$
- Vary $d = 0.2 - 0.5\ \mathrm{mm}$, keep $\gamma \approx 0.91$ constant
- At low frequencies, the lamination thickness has strong influence on the losses
- At very high frequencies, the lamination thickness has no influence on the losses

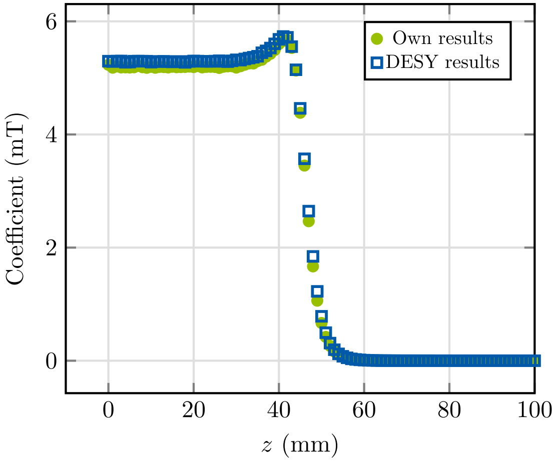
Simulation uses the same current for all frequencies !

Longitudinal Multipole Distribution (Static)

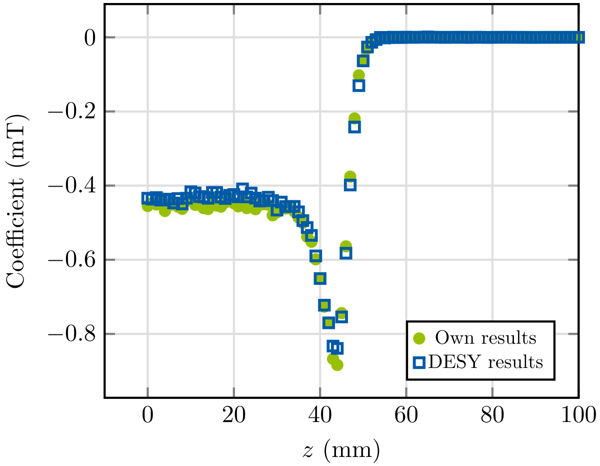
Dipole Coefficients



14-pole Coefficients



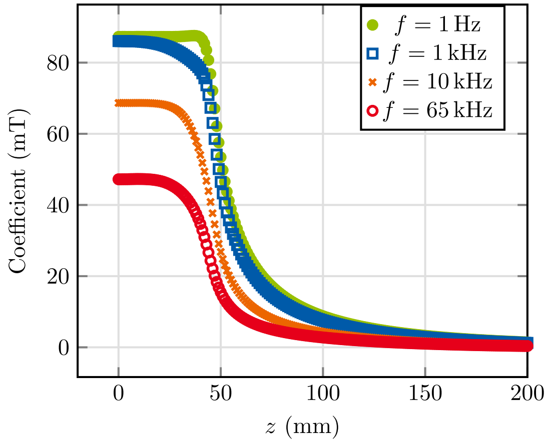
18-pole Coefficients



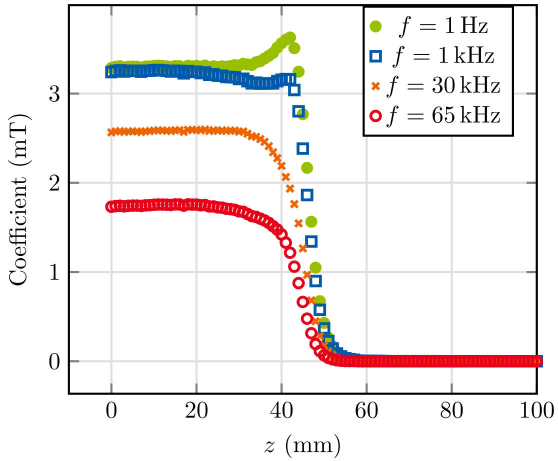
- Compute multipole coefficients along longitudinal axis of the magnet
- Comparison with DESY for static case ➔ good agreement

Longitudinal Multipole Distribution (Time-Harmonic)

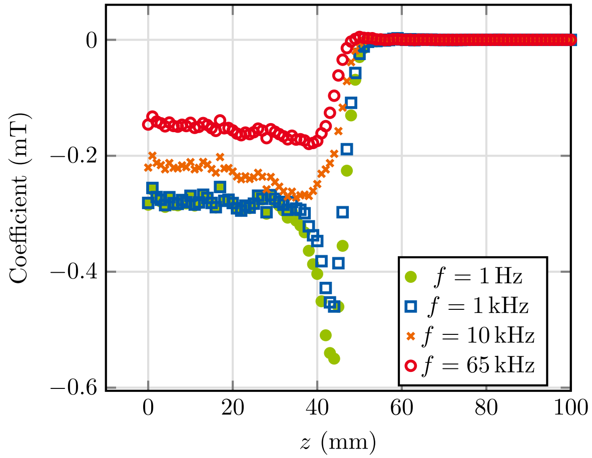
Dipole Coefficients



14-pole Coefficients



18-pole Coefficients



f (Hz)	Int. dipole (mT m)	Int. 14-pole (µT m)	Int. 18-pole (µT m)
1	11.6	316.4	−30.3
500	11.2	310.9	−29.7
1000	10.7	300.4	−28.6
10000	7.6	229.0	−21.5
30000	6.1	184.4	−17.2
65000	5.0	150.3	−13.9

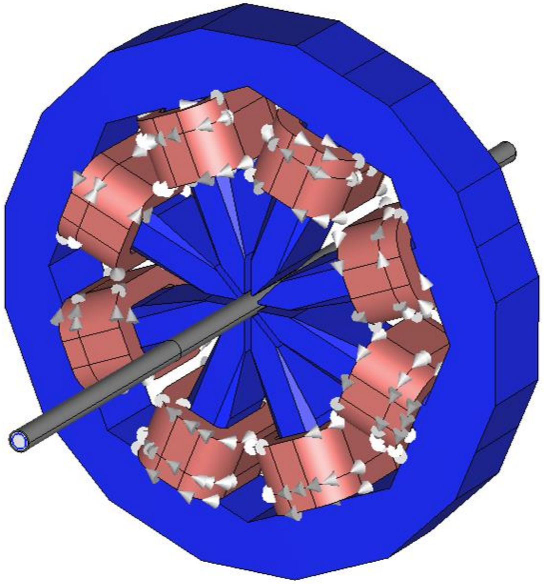
- **Updated # turns & current:**

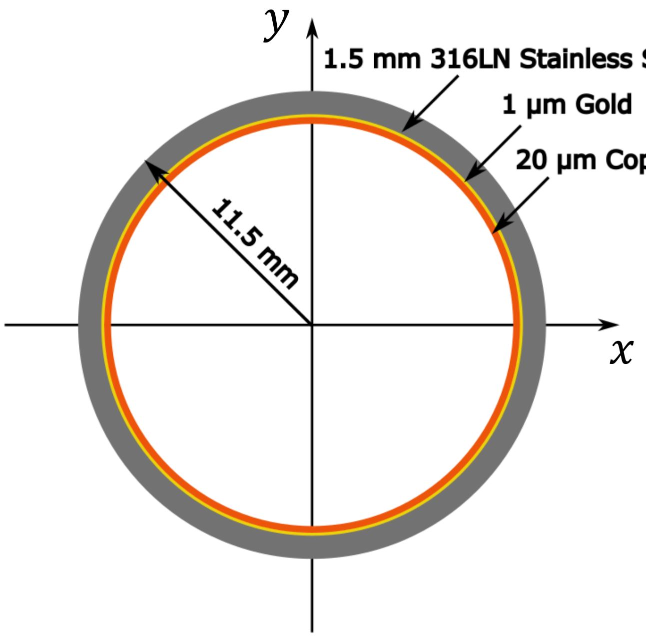
➔ Main coils: 65 turns, 15 A
➔ Aux. coils: 27 turns, 15 A

- 65 kHz vs. 1 Hz:

➔ Int. dipoles: −57 %
➔ Int. 14-poles: −52 %
➔ Int. 18-poles: −54 %

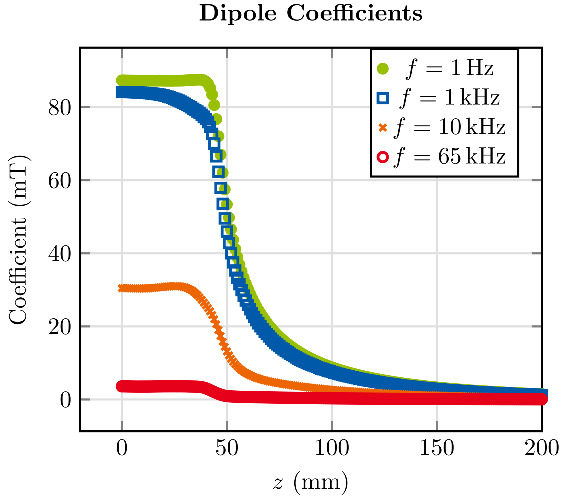
Realistic Model With Beam Pipe

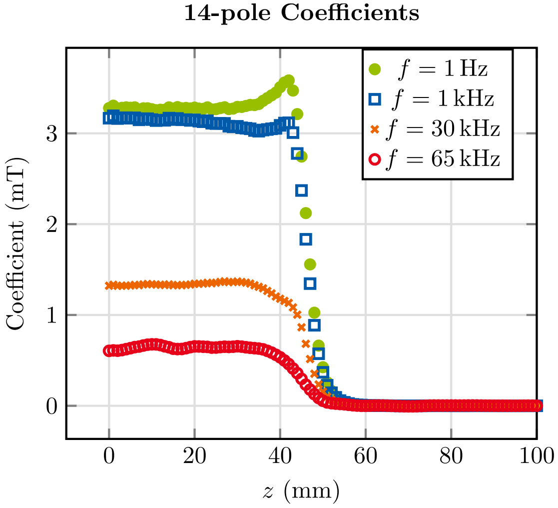


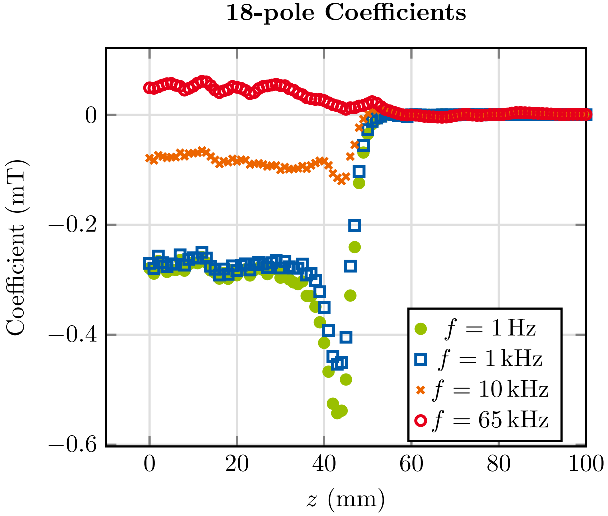


- Round beam pipe consisting of austenitic stainless steel layer, a thin copper layer, and an even thinner gold layer in between
- Longitudinal extent of beam pipe: $z = -500\,\mathrm{mm} \dots 500\,\mathrm{mm}$

Longitudinal Multipole Distribution


Dipole Coefficients


14-pole Coefficients


18-pole Coefficients

f (Hz)	Int. dipole (mT m)	Int. 14-pole (µT m)	Int. 18-pole (µT m)
1	11.5	313.3	−30.6
500	11.1	306.7	−29.7
1000	10.5	292.7	−28.1
10000	3.6	122.6	−8.3
30000	1.1	73.3	1.1
65000	0.4	57.4	4.3

- General shape similar to model without beam pipe
- 65 kHz vs. 1 Hz:

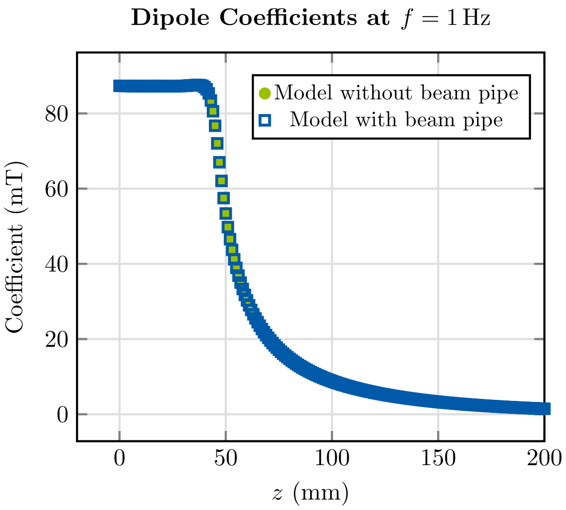
➔Int. dipoles: −97 % (−57 %)

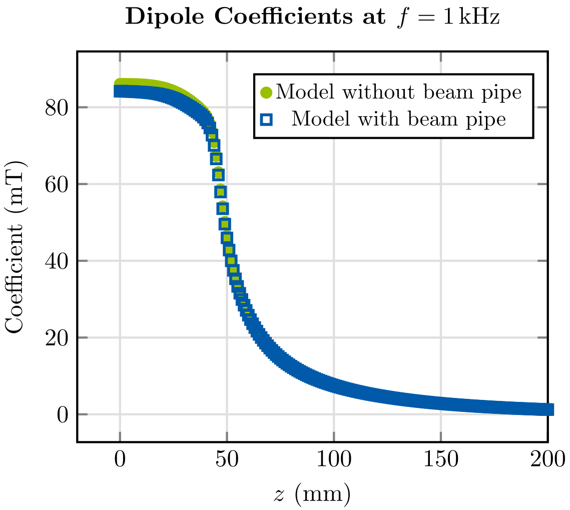
➔Int. 14-poles: −82 % (−52 %)

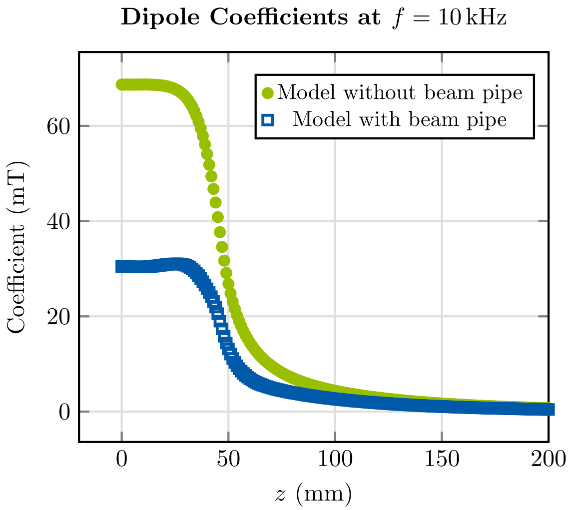
➔Int. 18-poles change sign (−54 %)

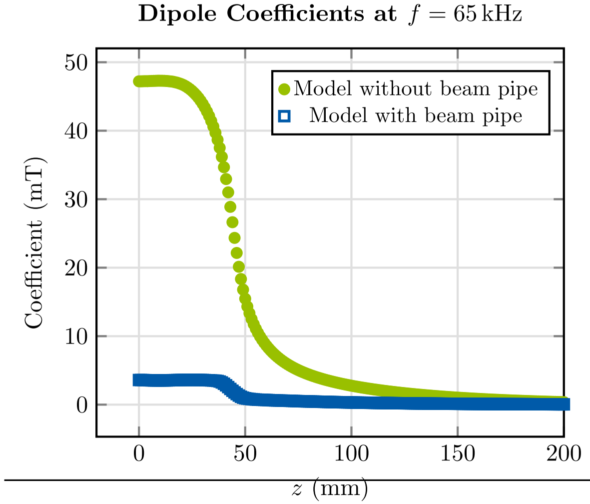
Without beam pipe

Model With Beam Pipe vs. Without Beam Pipe


Dipole Coefficients at $f = 1\,\mathrm{Hz}$


Dipole Coefficients at $f = 1\,\mathrm{kHz}$


Dipole Coefficients at $f = 10\,\mathrm{kHz}$


Dipole Coefficients at $f = 65\,\mathrm{kHz}$

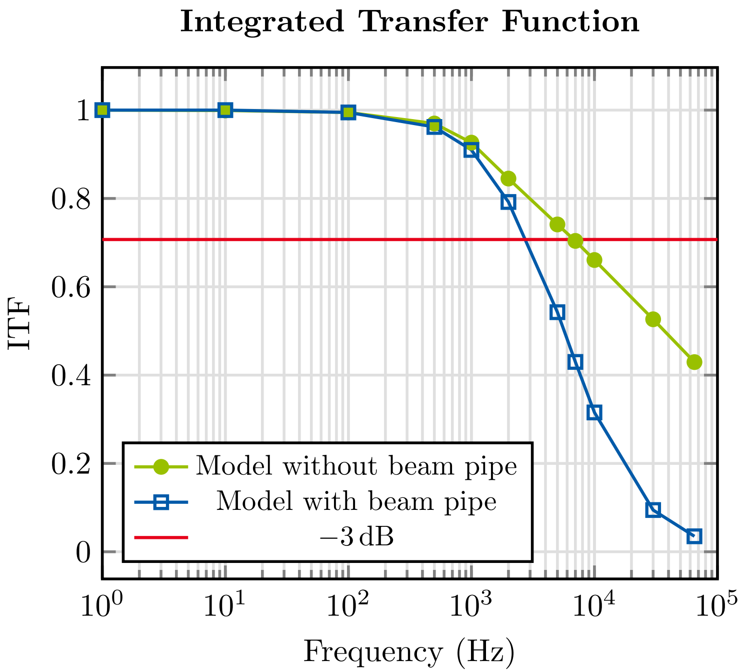
- Up to $f \approx 1\,\mathrm{kHz}$ only minor differences between two models

- For $f >> 1\,\mathrm{kHz}$: Strong attenuation of dipole field due to eddy currents in beam pipe

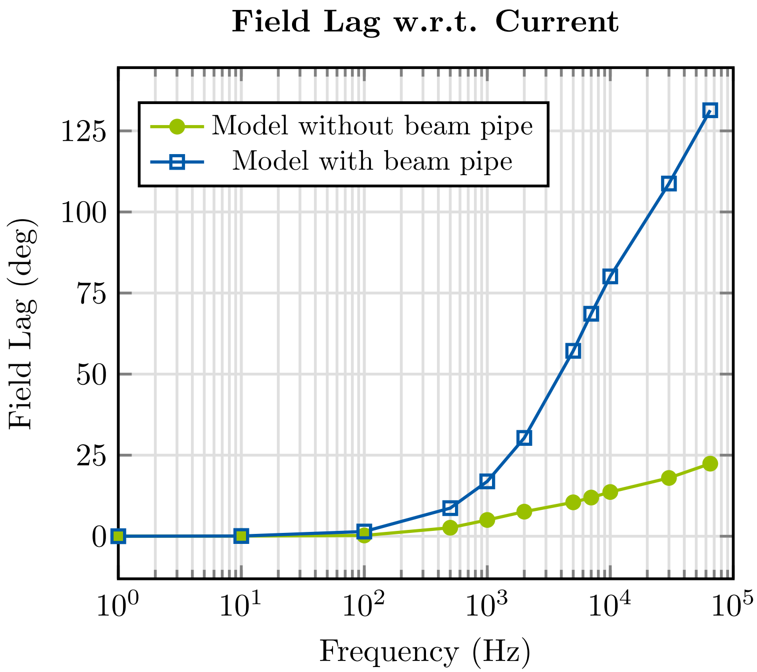
- At higher frequencies, beam pipe leads to greater effective length of the magnet

Model With Beam Pipe vs. Without Beam Pipe

Integrated Transfer Function



Field Lag w.r.t. Current



$$\mathrm{ITF}(f) = \frac{\int_l B_1(z,f)\,\mathrm{d}z}{\int_l B_1(z,f = 1\mathrm{Hz})\,\mathrm{d}z}$$

	Model without beam pipe	Model with beam pipe
3 dB bandwidth	7 kHz	3 kHz
Max. field lag	22°	132°

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Conclusion/Outlook

- Validation of homogenization technique using **toy model**
  - ➔ Good approximation of multipoles and power losses
  - ➔ Simulation time reduced from several hours to a few minutes
- Application to **realistic model without beam pipe** (DC up to $65\ \mathrm{kHz}$)
  - Study of power losses for different lamination thicknesses
  - Study of longitudinal multipole distributions
- Application to **realistic model with beam pipe** (DC up to $65\ \mathrm{kHz}$)
  - Study of longitudinal multipole distributions
  - Comparison to model without beam pipe
- **Next steps**: Continue study of realistic model with beam pipe, investigate model with thinner beam pipe and different yoke materials

References

[1] PETRA IV Conceptual Design Report.

[2] H. De Gersem, S. Vanaverbeke, and G. Samaey, “ Three-Dimensional-Two-Dimensional Coupled Model for Eddy Currents in Laminated Iron Cores,” *IEEE. Trans. Magn*., vol. 48, no. 2, pp.815 – 818, Feb. 2012.

[3] P. Dular et al., “A 3-D Magnetic Vector Potential Formulation Taking Eddy Currents in Lamination Stacks Into Account*,” IEEE Trans. Magn*., vol. 39, no. 3, pp. 1424-1427, May 2003.

[4] J. Gyselinck and P. Dular, “A Time-Domain Homogenization Technique for Laminated Iron Cores in 3-D Finite-Element Models,” *IEEE Trans. Magn.,* vol. 40, no. 2, pp. 856 - 859, Mar. 2004.

[5] L. Krähenbühl et al., “Homogenization of Lamination Stacks in Linear Magnetodynamics,” *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 912 - 915 Mar. 2004.

[6] K. Wille, *Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen*. Stuttgart, Germany: Teubner, 1992.

[7] S. Koch, “Quasistatische Feldsimulationen auf der Basis von Finiten Elementen und Spektralmethoden in der Anwendung auf supraleitende Magnete,” Ph.D. dissertation, TU Darmstadt, 2009.

[8] J. Lammeraner and M. Stafl*, Eddy Currents*. Iliffe books, 1996.